ISSN 1311-0829

ГОДИШНИК НА ТЕХНИЧЕСКИ УНИВЕРСИТЕТ-СОФИЯ

том 67, книга 3, 2017

PROCEEDINGS OF TECHNICAL UNIVERSITY OF SOFIA

Volume 67, Issue 3, 2017

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ISSN 1311-0829

СЪДЪРЖАНИЕ том 67, книга 3

CONTENTS Volume 67, Issue 3

Author's Index - Volume 67, Issue 3

PROCEEDINGS OF TECHNICAL UNIVERSITY OF SOFIA

Volume 67 Issue 3 (2017)

pages	articles	authors
150	15	18

СРАВНЕНИЕ НА ОСНОВНИ ХАРАКТЕРИСТИКИ НА ДВИГАТЕЛИ НА STIRLING ПРИ ВАРИРАНЕ НА ПАРАМЕТРИТЕ

Цветелина Петрова

Резюме: *Настоящата работа представя резултати от моделни изследвания относно работата на два типа двигатели на Stirling (с кинематично свързани бутала и хибриден двигател на Stirling-Ringbom) при промяна на режимни параметри. Извършен е сравнителен анализ на основни показатели, характеризиращи работата на изследваните машини.*
Ключови думи: *моделиране на двигател на Stirling, механична мощност, коефициент на полезно действие*

COMPARISON OF THE MAIN CHARACTERISTICS OF STIRLING ENGINES AT VARYING THE PARAMETERS

Tsvetelina Petrova

Abstract: *This work presents the modeling results of the behavior of two types of Stirling engines, a Stirling engine with cinematically linked pistons and a hybrid Stirling-Ringbom engine, at changing of regime parameters. A comparative analysis of the main indicators characterizing the performance of the investigated machines has been done.*
Keywords: *modeling of a Stirling engine, mechanical power, efficiency*

ВЪВЕДЕНИЕ

В наши дни все повече усилия са насочени към изследването на нови възможности за получаване на електрическа енергия – от двигател на Stirling, работещ с практически всеки източник на топлина, до турбини, задвижвани от морските или океанските вълни [1]. Двигателят на Stirling има много разновидности [2, 3], които могат да бъдат класифицирани по различни показатели. Единият от тях е начина на свързване на буталата: с кинематично свързани бутала, хибриден двигател на Stirling-Ringbom, със свободни бутала (Free Piston Stirling Engine) [4].

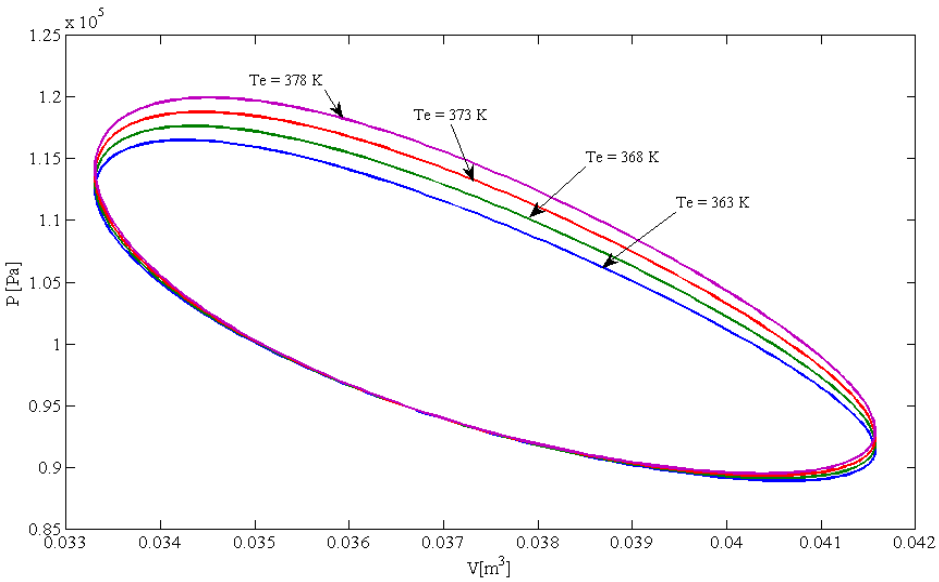
Целта на настоящата работа е да се да се сравнят основни работни характеристики на два типа двигатели на Stirling – хибриден двигател на Stirling- Ringbom и двигател на Stirling с кинематично свързани бутала при промяна на условията на работа. Двата типа изследвани двигатели са с еднакви конструктивни и режимни параметри. Разликата е в свързването на двете бутала – при двигател на Stirling с кинематично свързани бутала и двете бутала (силовото бутало и дисплейсъра)

ISSN 1311-0829

са свързани към маховик, докато при хибридния двигател на Stirling-Ringbom силовото бутало е свързано към маховик, а дисплейсърът (изместващото бутало) се движи свободно. За постигане на поставената цел е изследвана работата на двете машини при варианe на един от следните параметри: температурата на горещия температурен източник T_e, масата на работния флуид M и атмосферното налягане B. Симулациите са извършени с помощта на създадени модели, подробно описани в [5, 6, 7].

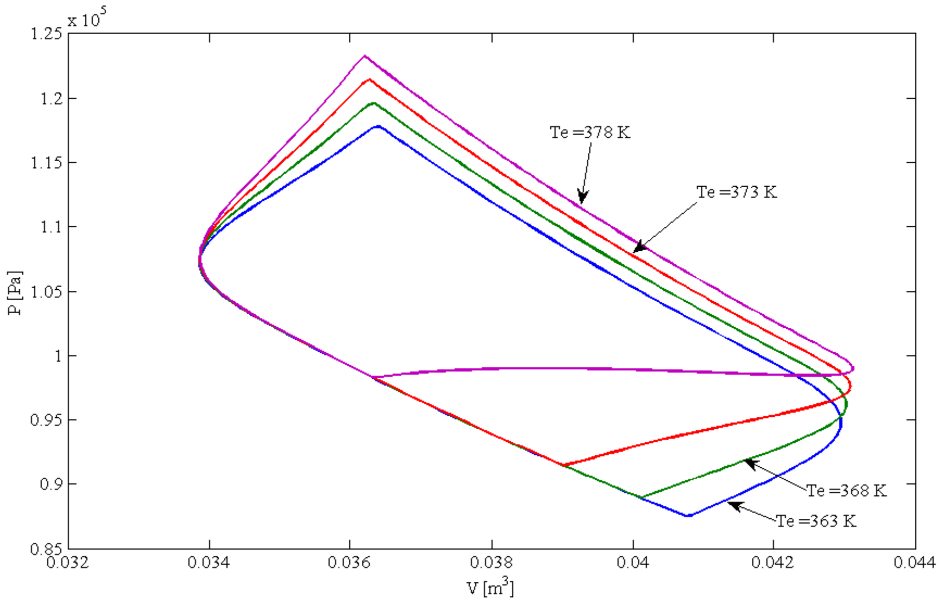
СРАВНЕНИЕ НА ПОЛУЧЕНИТЕ РЕЗУЛТАТИ

Извършено е изследване за влиянието на температурата на горещия температурен източник T_e върху работата на двигател на Stirling-Ringbom и двигател на Stirling с кинематично свързани бутала. За целта е моделирана работата на двигателите при четири стойности на T_e = 363K, 368K, 373K и 378K. Получените p-V диаграми за двигател на Stirling с кинематично свързани бутала са показани на фиг.1. С повишаване на T_e се увеличава извършената полезна работа по време на цикъла.



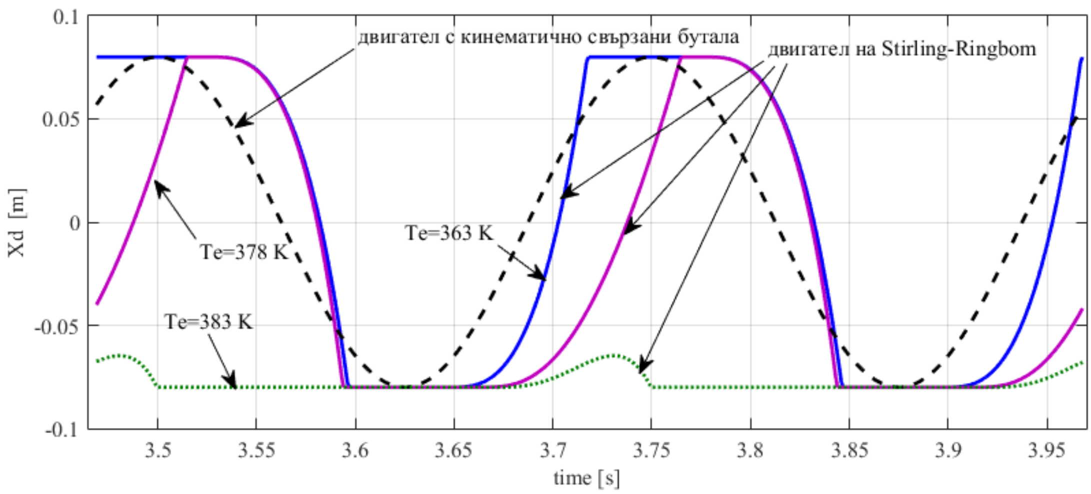
Фиг.1. Работни p-V диаграми на двигател на Stirling с кинематично свързани бутала при различни стойности на T_e

Резултатите за двигател на Stirling-Ringbom са дадени в графичен вид на фиг.2. Наблюдава се промяна в работната p-V диаграма за различните стойности на T_e. Затворените криви показват полезната работа, получена от цикъла. Постепенно, с нарастване на температурата, режимът на работа на изследвания двигател се променя, става неустойчив, а получаваната полезна работа започва да намалява. Това се дължи на промените в движението на дисплейсъра по време на работа на машината.



Фиг.2. Работни p-V диаграми на двигател на Stirling-Ringbom при различни стойности на T_e

На фиг.3 са показани положенията на дисплейсъра X_d на двигателя на Stirling-Ringbom, както и на двигател на Stirling с кинематично свързани бутала, за различни стойности на T_e. Положението на дисплейсъра на двигател на Stirling с кинематично свързани бутала е изобразено с черна прекъсната линия. Независимо от температурата, движението на изместващото бутало на този тип двигател остава непроменено. При хибридния двигател времето, през което дисплейсърът се задържа в крайно горно и крайно долно положение, се променя съществено в зависимост от температурата T_e. На фиг.3 това задържане отговаря на хоризонталните участъци в графиките за положението на X_d при температури T_e = 363 K и T_e = 378 K. При температура T_e = 383 K дисплейсърът вече не се движи от горната до долната си мъртви точки. При такава стойност на температурата се нарушава нормалният режим на работа на хибридния двигател.



Фиг.3. Положение на дисплейсъра X_d на изследваните двигатели при работа с различни T_e

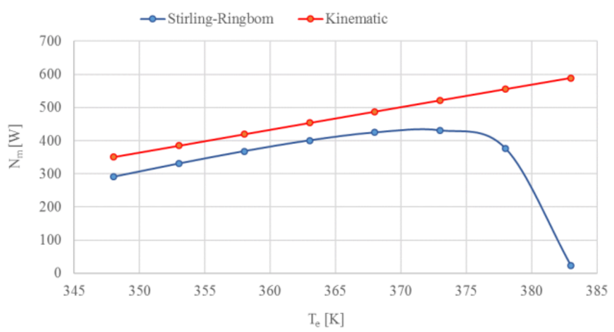
Представени са и резултати в графичен вид от направените моделни изследвания, извършени при вариране около стойностите на един от следните параметри, при които и двата двигателя се характеризират с устойчива работа: M = 0,041625 kg, T_e = 363 K, атмосферно налягане B = 1,01625e5 Pa. Конкретните стойности на променяните параметри са дадени в табл.1.

Таблица 1.

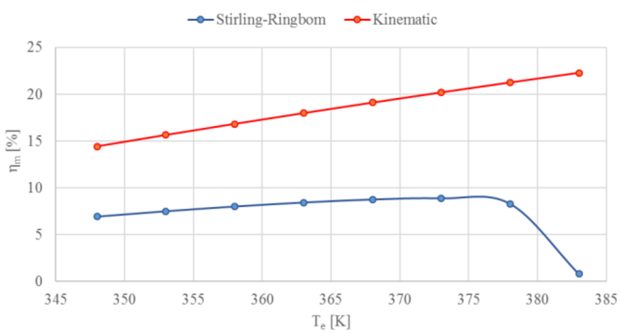
Стойности на параметрите, с които са проведени изследванията

T_e [K]	348	353	358	**363**	368	373	378	383
M [kg]	0,0406	0,041	0,043	**0,0416**	0,042	0,0423	0,0426	0,043
B [Pa]	98 325	99 325	100 325	**101 325**	102 325	103 325	103 800	

Сравнението на симулационните резултати за влиянието на температурата на горещия температурен източник T_e върху механичната мощност N_m на изследваните двигатели е дадено в графичен вид на фиг.4. При еднакви геометрични параметри, двигателят на Stirling-Ringbom е съществено ограничен по отношение на диапазона на изменение на температурата на горещия температурен източник T_e. За разглеждания случай максимумът на мощността е при T_e = 373 K. Топлинната мощност на двигателя на Stirling с кинематично свързани бутала (Kinematic) расте с повишаване на T_e. Този тип двигател позволява много по-големи възможности за промяна на T_e.
На фиг.5 е показана промяната на механичния коефициент на полезно действие η_m на двата типа двигатели при промяна на температурата на горещия температурен източник T_e. С повишаване на този режимен параметър механичният коефициент на полезно действие на двигателя на Stirling с кинематично свързани бутала нараства, докато η_m за хибридния двигател рязко намалява след T_e = 378 K.



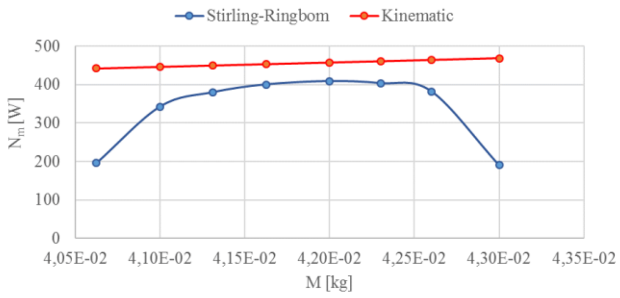
Фиг.4. Механична мощност на двигателите във функция на T_e



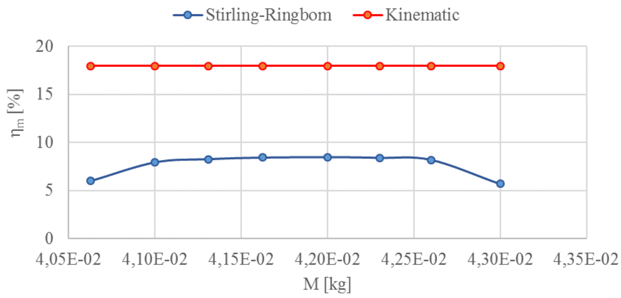
Фиг.5. Механичен КПД на двигателите във функция на T_e

При по-високи температури T_e се повишава и налягането на работния флуид. Повишеното налягане в двигателя не позволява извършването на пълен ход на изместващото бутало към долната му мъртва точка, както и неговото задържането в крайно долно и крайно горно положение. Такъв режим влияе отрица-

телно на работата и производителността на машината.
Двете машини са изследвани за влиянието на масата на работния флуид M върху механичната мощност и върху механичния коефициент на полезно действие. На фиг.6 е показано влиянието на M върху N_m. При една и съща стойност на масата на флуида получаваната механична мощност от двигател на Stirling с кинематично свързани бутала (Kinematic) е по-висока. Коефициентът на полезно действие на този тип двигател също е по-висок в сравнение с η_m на двигателя на Stirling-Ringbom (фиг.7).

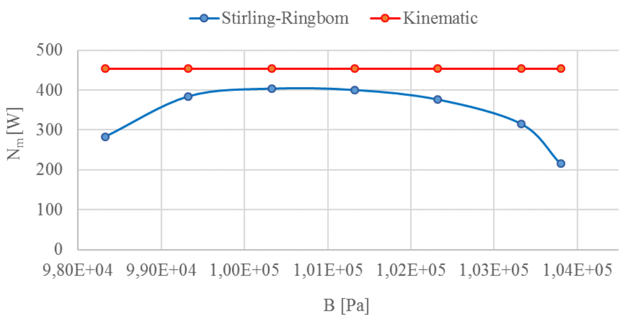


Фиг.6. N_m във функция на масата на работния флуид

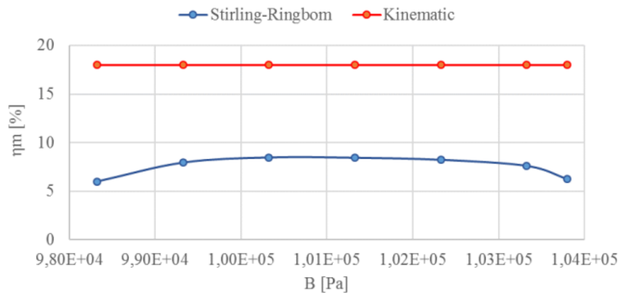


Фиг.7. η_m във функция на масата на работния флуид

Изследвано е влиянието на атмосферното налягане B върху получаваната механична мощност (фиг.8) и коефициента на полезно действие (фиг.9) на двете машини. И двете величини, характеризиращи работата на двигателите, са с по-високи стойности за двигател на Stirling с кинематично свързани бутала (Kinematic).



Фиг.8. N_m във функция на атмосферното налягане B



Фиг.9. η_m във функция на атмосферното налягане B

ЗАКЛЮЧЕНИЕ

Въз основа на извършените моделни изследвания могат да бъдат направени следните заключения:

- Получената механична мощност от двигател на Stirling с кинематично свързани бутала е с по-високи стойности в сравнение с тази, получена от двигател на

Stirling-Ringbom;

- Двигателят на Stirling с кинематично свързани бутала е с по-висок коефициент на полезно действие в сравнение с двигател на Stirling-Ringbom;

- Двигателят на Stirling с кинематично свързани бутала в сравнение с хибридния двигател на Stirling-Ringbom е с възможност за устойчива работа при значително по-голям диапазон на промяна на температурата на горещия температурен източник T_e, на масата на работния флуид M и на атмосферното налягане B. Поради специфичната си конструкция двигателят на Stirling-Ringbom е силно чуствителен към промяна на параметрите и незначителни за двигателят на Stirling с кинематично свързани бутала промени в условията на работа водят до загуба на работоспособност при хибридния двигател.

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Постъпила на 10.05.2017 **Рецензент:** доц. д-р Росица Т. Величкова

Годишник на Технически Университет - София, том 67, книга 3, 2017
Proceedings of the Technical University of Sofia, Volume 67, Issue 3, 2017

ОТНОСНО АНАЛИЗА НА ПРИРОДНИТЕ БЕДСТВИЯ

Росица Величкова, Детелин Марков, Искра Симова, Георги Бърдаров, Цветелина Петрова, Захари Кетипов

Резюме: *В работата се разглежда въпроса за хармонизирането на класификацията на типовете природни бедствия между няколко глобални бази данни с цел подобряване на качеството и надеждността на базата данни за бедствията в България. Представена е класификация на природните бедствия и тяхното въздействие върху хората. Представен е и кратък анализ на характера и разпределението на природните бедствия в света за периода 2000-2015 г., както и за въздействието им върху човечеството.*
Ключови думи: *класификация, природни бедствия, влияние върху човечеството*

ON THE ANALYSIS OF NATURAL HAZARDS

Rositsa Velichkova, Detelin Markov, Iskra Simova, Georgi Burdarov, Tsvetelina Petrova, Zahari Ketipov

***Abstract*: *This paper deals with harmonization of the natural disasters category classification between some global disaster databases with the aim to improve the quality and reliability of the Bulgarian disaster database. A classification of the natural hazards is presented and their impact on people is specified. It is presented as well a brief discussion on the nature and distribution of disaster events around the globe during the period 2000 – 2015 as well as on its impact on mankind.*
Key words: *classification, natural hazards, impacts on human*

1. INTRODUCTION

According to the Red Cross definition, [7], disaster is a ”sudden, calamitous event that seriously disrupts the functioning of a community or society and causes human, material, and economic or environmental losses that exceed the community’s or society’s ability to cope using its own resources”. Though often caused by nature, disasters can have human origins, [7]. ***A disaster occurs when a hazard impacts on vulnerable people,*** [7]. The combination of hazards, vulnerability and inability to reduce the potential negative consequences of risk results in disaster, [7].

Natural disasters are catastrophic events with atmospheric, geologic, and hydrologic origins. Disasters include earthquakes, volcanic eruptions, landslides, tsunamis, floods, and drought. Natural disasters can have rapid or slow onset, with serious health, social, and economic consequences. During the past two decades, natural disasters have killed millions of people, adversely affected the lives of at least 1 billion

ISSN 1311-0829

more people, and resulted in substantial economic damages, [1].

The aim of the present work is to propose a harmonized classification of natural hazards and their impact on the people, which will be used for establishing of the Bulgarian disasters database.

2. HIERARCHY OF DISASTER CATEGORIES

The harmonized classification of disasters is prepared on the basis of the information included in the databases described in [2, 5, 6]. It distinguishes two generic disaster groups: natural and technological disasters. The natural disaster group is divided into six disaster sub-groups: Biological, Geophysical, Meteorological, Hydrological, Climatological and Extraterrestrial. Each sub-group includes several disaster main types, each of them having some disaster sub-types. Harmonized classification of the natural disasters is presented in Table 1.

3. IMPACTS OF NATURAL HAZARDS

Typically, the number of reported victims of a disaster is composed by the number of those who are killed and those who are otherwise affected. In the period 2000 – 2015 the victims number of climatological, geophysical, hydrological and meteorological disasters in Africa is 17.9 million, in America this number is 21.1 million, in Asia is 278.4 million, in Europe 1.1 million, and in Oceania 0.1 million. More than 87% of the victims have been living in Asia.

On Figure 1 is visualized the distribution of the victims among the above mentioned disaster sub-groups for the same five geographical regions. The most of the victims in Africa are due to climatological disasters, while in all other regions most of the victims are due to hydrological disasters. The most of the victims worldwide, 37.9%, are due to hydrological disasters. In all regions, except Asia, the smallest are the victims of geophysical disasters.

Another striking indicator for characterization of the impact of disasters on the society is the number of the killed people (death toll, disaster fatalities). On Figure 2 is visualized the death toll distribution by type of disasters in the period 2000-2015 for four groups of countries – OECD countries (Organization for economic cooperation and development), the countries of Central and Eastern Europe and Commonwealth of independent states, Developing countries and the Least developed countries.

The total number of disaster fatalities in these groups of countries is as follows: OECD countries – 61929; Central and Eastern Europe and Commonwealth of independent states- 10412; Developing countries - 630106; Least developed countries – 254739.

The most of the fatalities for all those countries (43.4%) is due to earthquakes and tsunamis, another 23% are due to windstorms, 12.7% due to floods and 12.5% due to epidemics.

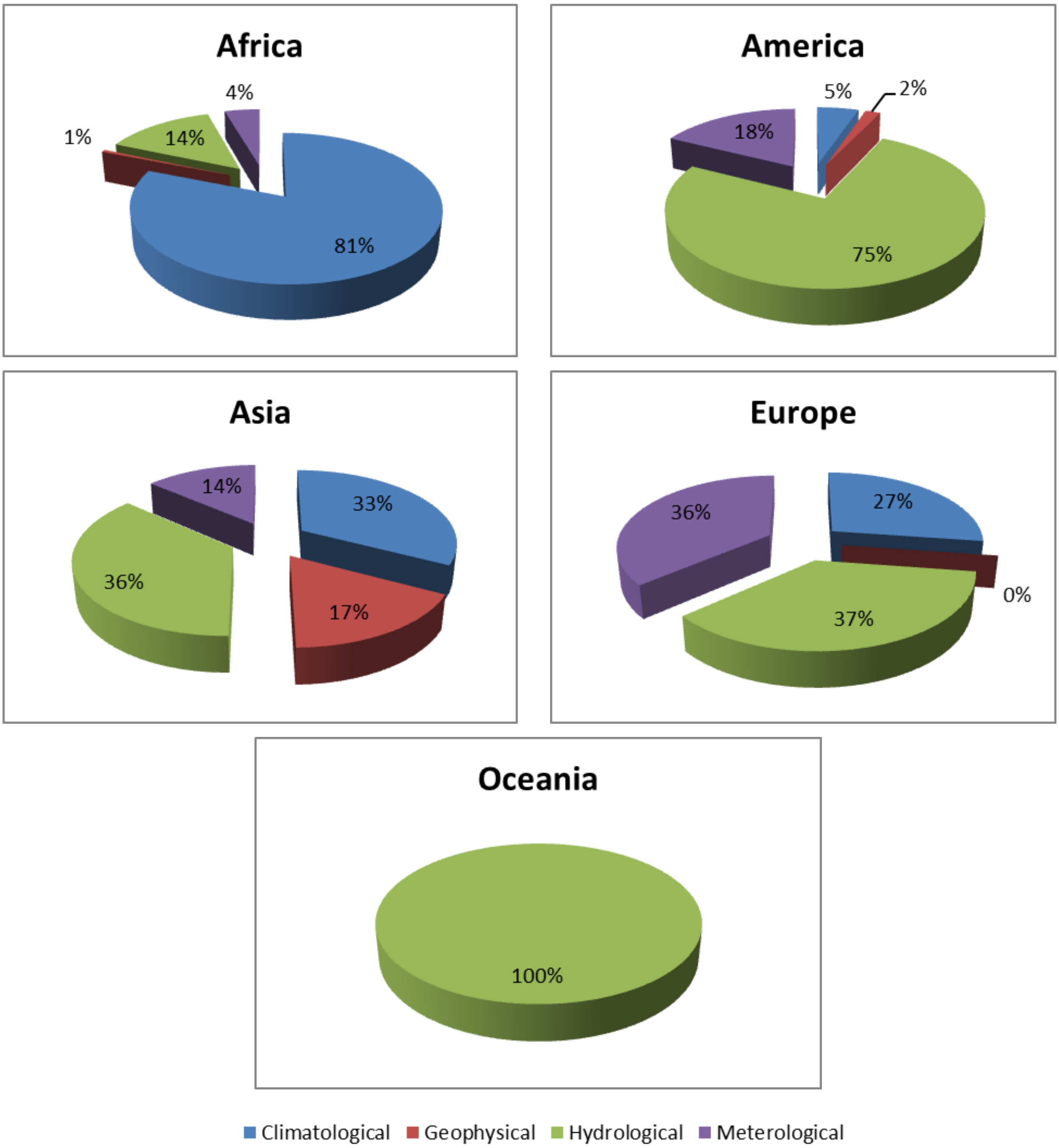

Figure 1. Natural hazards victims in the period 2000-2015

The most of the fatalities (77%) in the OECD countries are due to the drought (including extreme temperature). The other two main reasons for fatalities in this group of countries are earthquakes and tsunamis (9.5%), and windstorms (8.8%).

Drought is the most often reason for disaster fatalities in Central and Eastern Europe and the Commonwealth of independent states (29.9%). The other two main reasons for fatalities in this group of countries are floods (25.3%), and earthquakes and tsunamis (23.2%).

Earthquakes and tsunamis are the most often reasons for disaster fatalities in the group of Developing countries (63%). The other two main reasons for fatalities in this group of countries are floods (15.4%), and windstorms (10.4%).

Windstorms are the most often reasons for disaster fatalities in the group of the least developed countries (58.7%). The other two main reasons for fatalities in this group of countries are epidemic (27.7%), and flood (7.9%).

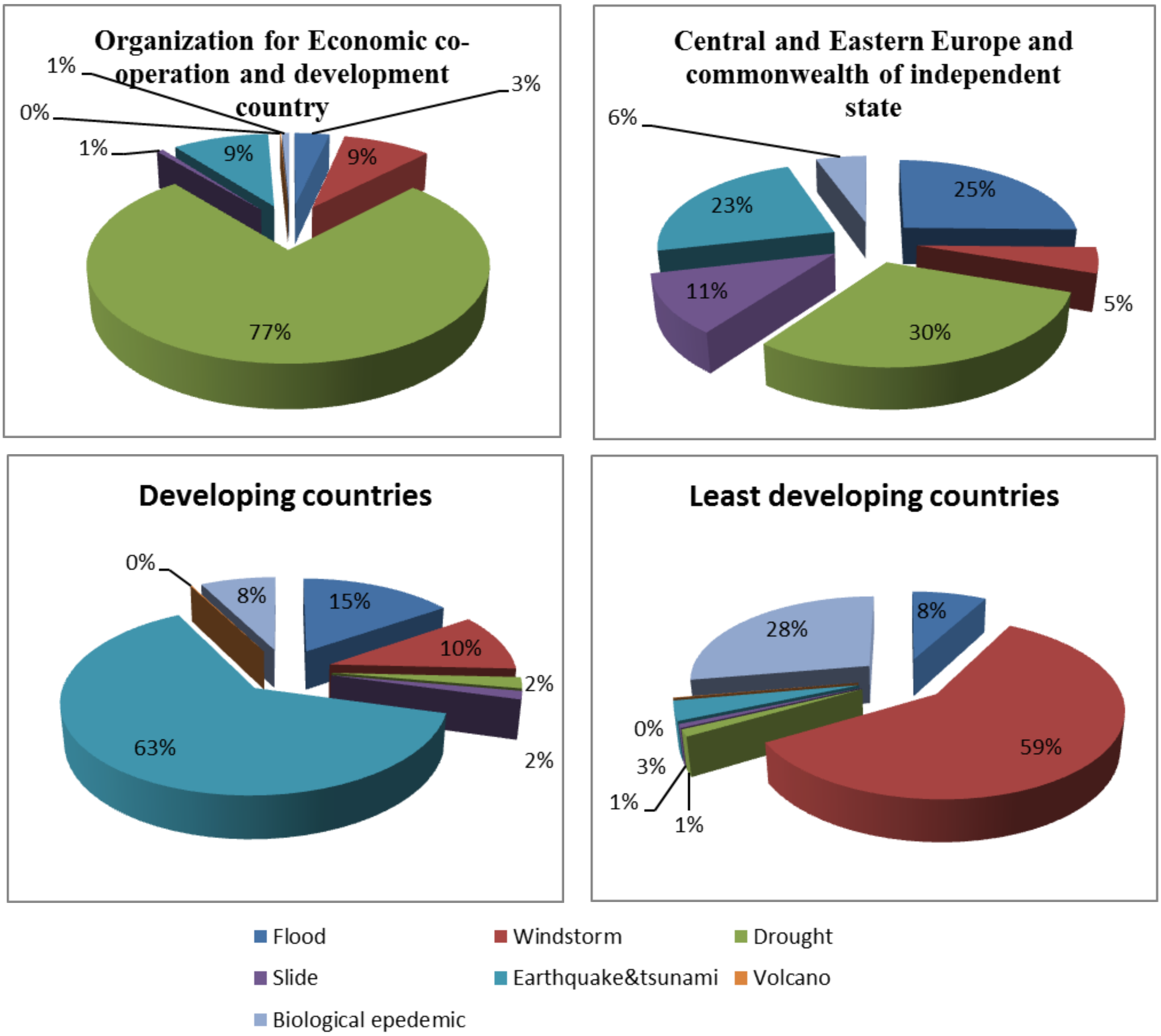

Figure 2. Natural disaster fatalities in the period 2000 - 2015

Disasters are usually classified according to their frequency and their impact on the society, measured by the number of victims and/or economic damage.

On Figure 3 the percentage of disasters of a certain type in five geographic regions is shown based on the data in the period 2000– 2015.

The total number of disasters in Africa is 1001, in America 1417, in Asia 2281, in Europe 1502, and in Oceania 212. The most of disaster events happens in Asia, followed by Europe, America, and Africa.

Table 1. Harmonized classification of the natural disasters

Disaster Group	Disaster Sub-group	Definition	Disaster Main Type	Disaster Sub-type	Disaster Sub-Sub type
Natural disaster	Geophysical	A hazard originating from solid earth. This term is used interchangeably with the term geological hazard	Earthquake	Ground Shaking	
				Tsunami	
			Volcano	Ash fall	
				Lahar	
				Pyroclastic flow	
				Lava flow	
			Mass movement (dry)	Rock fall	
				Avalanche	Snow avalanche
				Landslide	Mudslide
					Lahar
					Debris flow
				Subsidence	Sudden subsidence
					Long lasting subsidence
	Meteorological	A hazard caused by short-lived, micro to meso-scale extreme weather and atmospheric conditions that last from minutes to days	Storm	Extra tropical storm	
				Tropical storm	
				Convective storm	Derecho
					Hail
					Lighting/Thunderstorm
					Rain
					Tornado

					Sand/dust storm
					Wind storm/blizzard
					Storm/surge
					Wind
			Extreme temperature	Cold Wave	
				Heat wave	
				Severe winter conditions	Snow/Ice
					Frost/freeze
			Fog		
	Hydrological	A hazard caused by the occurrence, movement, and distribution of surface and subsurface freshwater and saltwater.	Flood	Storm surge/ coastal flood	
				General (river) flood	
				Flash flood	
				Ice jam flood	
			Mass movement (wet)	Rock fall	
				Landslide	
				Avalanche	Snow avalanche
					Debris avalanche
				Subsidence	Sudden subsidence
					Long-lasting subsidence
			Wave action	Rogue wave	
				Seiche	
	Climatological	A hazard caused by long-lived, meso -to macro-scale atmospheric processes ranging from intra-seasonal to multi-decadal climate variability.	Extreme temperature	Heat wave	
				Cold wave	Frost
				Extreme winter condition	Snow pressure
					Icing

					Freezing rain
					Debris avalanche
	Climatological	A hazard caused by long-lived, meso -to macro-scale atmospheric processes ranging from intra-seasonal to multi-decadal climate variability.	Drought	Drought	
			Wild fire	Forest fire	
				Land fire (grass, scrub, brush and etc.)	
			Glacial Lake Outburst		
	Biological		Epidemic	Viral Disease	
				Bacterial Disease	
				Parasitic Disease	
				Fungal Disease	
				Prion Disease	
			Insect infestation	Grasshopper	
				Locust	
			Animal stamped		
	Extraterrestrial		Impact	Airburst	
			Space weather	Energetic particle	
				Geomagnetic storm	
				Shock wave	

Based on the number of disaster events the world suffers the most from hydrological disasters (49%). In descending order, they are followed by the meteorological (28%), climatological (14%), and geophysical (9%) disasters.

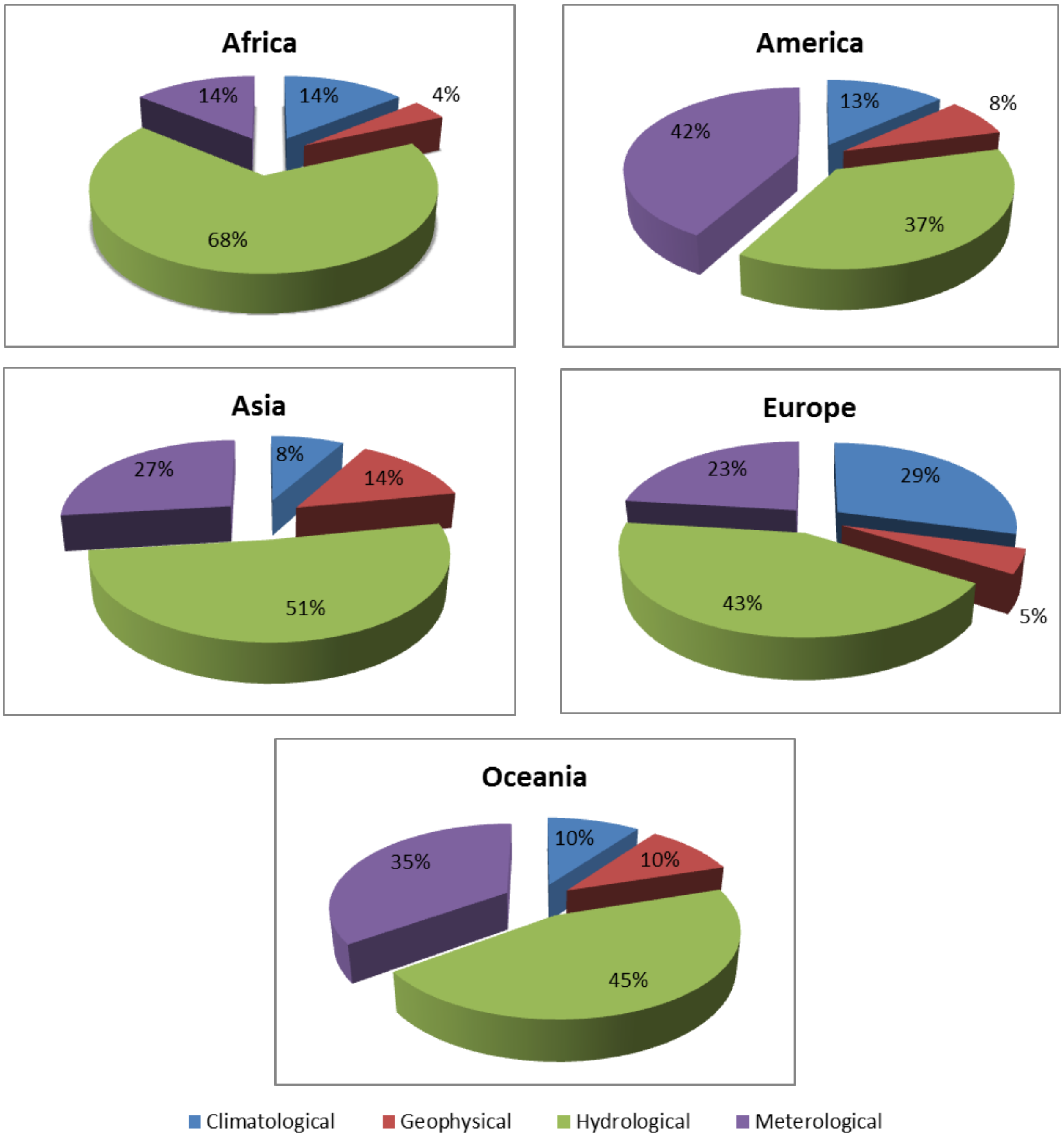

Figure 3. Results for disaster events worldwide in the period 2000 - 2015

In all continents, the most of the disasters have hydrological origin, except America, which suffers the most by disasters of meteorological origin (42%) followed by hydrological disasters (37%). For three of the continents the second most frequent type of disasters is those with meteorological origin. For America, this type of disasters is dominant while for Europe it is on the third place (23%) surpassed by the climatological disasters (29%). For all continents, except for Asia, the smallest number of disasters belongs to the geophysical group.

Figure 4 presents the worldwide economic damages, estimated in bill $, caused by disasters in the period 2000 – 2015. The total amount of economic damages due to disasters for this period in Africa is 16.7 bill $, in America – 798.4 bill $, in Asia - 1074 bill $, in Europe - 149 bill $, and in Oceania – 25.5 bill $.

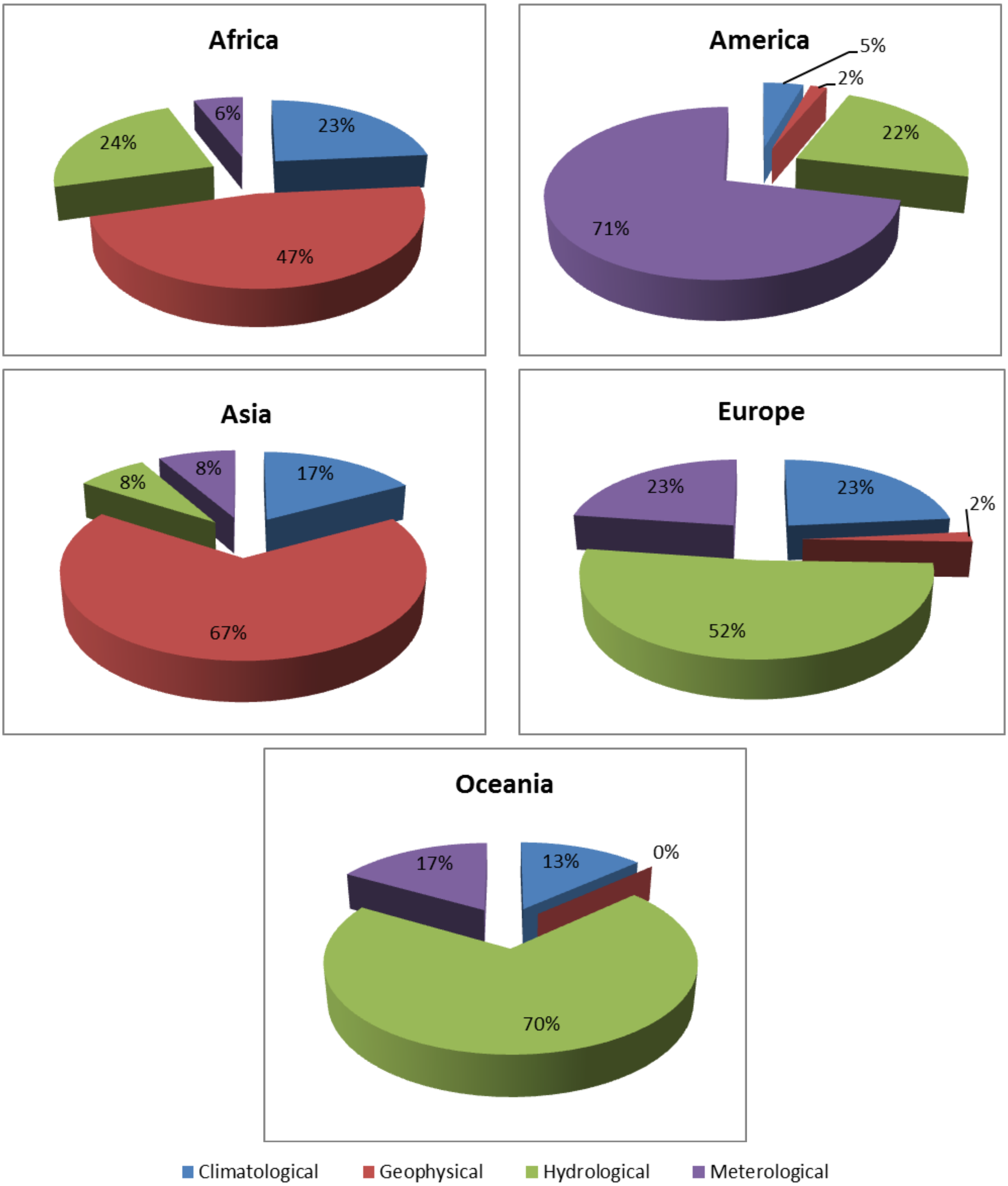

Figure 4. Results for economic damages worldwide caused by disasters in the period 2000 - 2015

Huge part of the economic damages is inflicted in Asia (52%) and America (39%).

The most of the economic damages worldwide are due to geophysical events (36%), followed by meteorological (34%), hydrological (17%) and climatological (13%) events. The reason for the biggest economic damages in each continent is different. In Africa (47%) and Asia (67%) the biggest economic damages are caused by geophysical disasters, in America (71%) by meteorological, while in Europe (52%) and Oceania (70%) by hydrological disasters.

4. CONCLUSION

The of harmonization of the disaster category classification between some global disaster databases as well as the impact of natural hazards on the society is an important contribution to the improvement of both the quality and reliability of the international disaster databases. Both, disaster classification and impacts on the society serve the international community, users and developers of databases at national or sub-national levels to improve their understanding for the management of disaster data. The harmonized disaster database demonstrates the importance to develop the capacity of information exchange, integration and comparability between disaster databases. Data stored in disasters database can be used only for statistical analysis of the disasters. In order to be applicable for risk analysis a disaster database must include geographical, demographic, urban and building construction data in a format convenient for processing by GIS applications. On the basis of the proposed harmonized classification of disasters events will be developed the Bulgarian disaster database.

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Received: November 09, 2017 **Reviewer:** Assoc. Prof. PhD G. Pichurov

Годишник на Технически Университет - София, том 67, книга 3, 2017
Proceedings of the Technical University of Sofia, Volume 67, Issue 3, 2017

ДИОФАНТОВО НЕРАВЕНСТВО СЪС СТЕПЕНИ НА ПРОСТИ ЧИСЛА ОТ СПЕЦИАЛЕН ВИД

Стоян Димитров

Резюме: *Нека N е достатъчно голямо реално число. В тази статия ние доказваме, че за всяко фиксирано $0 < c < 4/21$ и произволно малко $\varepsilon > 0$, дифантовото неравенство*

$$|p_1^c + p_2^c + p_3^c - N| < \varepsilon$$

има решение в прости числа p_1, p_2, p_3, такива че за всяко $i \in \{1, 2, 3\}$, $p_i + 2$ има най-много 10 прости делителя.

Ключови думи: *Тегла на Розер, векторно решето, диофантово неравенство.*

ON A DIOPHANTINE INEQUALITY WITH PRIME POWERS OF A SPECIAL TYPE

Stoyan Dimitrov

Abstract: *Assume that N is a sufficiently large real number. In this paper we prove that for any fixed $0 < c < 4/21$ and a small constant $\varepsilon > 0$, the diophantine inequality*

$$|p_1^c + p_2^c + p_3^c - N| < \varepsilon$$

is solvable in primes p_1, p_2, p_3 such that, for each $i \in \{1, 2, 3\}$, $p_i + 2$ has at most 10 prime factors.

Keywords: *Rosser's weights, vector sieve, diophantine inequality.*

1 Introduction and statements of the result.

In 1992 Tolev [7] proved that the diophantine inequality

$$|p_1^c + p_2^c + p_3^c - N| < \varepsilon \tag{1}$$

has a solution that for any fixed $1 < c < 15/14$ and a small constant $\varepsilon > 0$. The interval $1 < c < 15/14$ was subsequently improved by several authors [2], [4], [5], [1].

ISSN 1311-0829

In this paper we consider for first time the inequality (1) for $0 < c < 1$. Let P_l is a number with at most l prime factors. Using the vector sieve method we shall prove the following

Theorem 1 *Let $0 < c < 4/21$. There exists a number $N_0(c) > 0$ such that for each real number $N > N_0(c)$ and for an arbitrarily large A the inequality*

$$|p_1^c + p_2^c + p_3^c - N| < \left[\log \frac{2}{3}\left(\frac{N}{3}\right)^{1/c}\right]^{-A}$$

has a solution in prime numbers p_1, p_2, p_3 such that

$$p_1 + 2 = P'_{10}\,, \quad p_2 + 2 = P''_{10}\,, \quad p_3 + 2 = P'''_{10}\,.$$

By choosing the parameters in a different way we may obtain other similar results, for example $0 < c < 3/4\,,\ p_i + 2 = P_{47}\,, i = 1, 2, 3\,.$ Obviously the enlargement of the range for c leads to increase of the number of the prime factors of $p_i + 2$.

2 Notations and some lemmas.

The relation $f(x) \ll g(x)$ means that $f(x) = \mathcal{O}(g(x))$. For positive A and B we write $A \asymp B$ instead of $A \ll B \ll A$. As usual $\varphi(n)$ and $\mu(n)$ denote respectively, Euler's function and Möbius' function. Let (m_1, m_2) be the greatest common divisor of m_1, m_2. Instead of $m \equiv n \pmod k$ we write for simplicity $m \equiv n\,(k)$. As usual, $[y]$ denotes the integer part of y, $e(y) = e^{2\pi \imath y}$. Let c be an fixed real number such that $0 < c < 4/21$, N be an sufficiently large number and $A > 1$ be an arbitrary large number.
Denote

$$X = \frac{2}{3}\left(\frac{N}{3}\right)^{1/c}; \tag{2}$$

$$\vartheta = \frac{1}{(\log X)^A}; \tag{3}$$

$$\tau = \frac{(\log X)^{4A+176}}{X^c}; \tag{4}$$

$$K = \frac{\log^2 X}{\vartheta}; \tag{5}$$

$$z = X^\beta, \ \ 0 < \beta < 1/10; \tag{6}$$

$$D = X^{2/7}; \tag{7}$$

$$P(z) = \prod_{2 < p \le z} p\,. \tag{8}$$

The letter β will be specified latter.

Let $\lambda^{\pm}(d)$ be the lower and upper bounds Rosser's weights of level D, hence

$$|\lambda^{\pm}(d)| \leq 1, \quad \lambda^{\pm}(d) = 0 \quad \text{if} \quad d \geq D \quad \text{or} \quad \mu(d) = 0\,.$$

Lemma 1 *Let $\vartheta \in \mathbb{R}$ and $k \in \mathbb{N}$. There exists a function $\upsilon(y)$ which is k times continuously differentiable and such that*

$$\begin{aligned} &\upsilon(y) = 1 && \textit{for} && |y| \leq 3\vartheta/4\,; \\ &0 \leq \upsilon(y) < 1 && \textit{for} && 3\vartheta/4 < |y| < \vartheta\,; \\ &\upsilon(y) = 0 && \textit{for} && |y| \geq \vartheta\,. \end{aligned}$$

and its Fourier transform

$$\Upsilon(x) = \int\limits_{-\infty}^{\infty} \upsilon(y) e(-xy)\, dy$$

satisfies the inequality

$$|\Upsilon(x)| \leq \min\left(\frac{7\vartheta}{4}, \frac{1}{\pi|x|}, \frac{1}{\pi|x|}\left(\frac{k}{2\pi|x|\vartheta/8}\right)^k\right).$$

Proof. This is Lemma 1 of Tolev [7]. □

Lemma 2 *We have*

$$\int\limits_X^{2X}\int\limits_X^{2X}\int\limits_X^{2X} \upsilon(y_1^c + y_2^c + y_3^c - N)\, dy_1 dy_2 dy_3 \gg \vartheta X^{3-c}\,.$$

Proof. Denote the above integral by $B(X)$. By the definition of $\upsilon(y)$ we obtain

$$B(X) \geq \int\limits_X^{2X}\int\limits_X^{2X}\int\limits_{\substack{X \\ |y_1^c+y_2^c+y_3^c-N|<3\vartheta/4}}^{2X} dy_1 dy_2 dy_3 \geq \int\limits_{\lambda X}^{\mu X}\int\limits_{\lambda X}^{\mu X}\left(\int\limits_{\mathfrak{M}} dy_3\right) dy_1 dy_2\,,$$

where λ and μ are real numbers such that

$$1 < \left(\frac{3^{c+1}}{2^{c+1}} - 2^{c-1}\right)^{1/c} < \lambda < \mu < \left(\frac{3^{c+1}}{2^{c+1}} - \frac{1}{2}\right)^{1/c} < 2$$

and

$$\begin{aligned} \mathfrak{M} &= [X, 2X] \cap [(N - 3\vartheta/4 - y_1^c - y_2^c)^{1/c},\, (N + 3\vartheta/4 - y_1^c - y_2^c)^{1/c}] \\ &= [(N - 3\vartheta/4 - y_1^c - y_2^c)^{1/c},\, (N + 3\vartheta/4 - y_1^c - y_2^c)^{1/c}]\,. \end{aligned}$$

Thus by the mean-value theorem we get

$$B(X) \gg \vartheta \int\limits_{\lambda X}^{\mu X}\int\limits_{\lambda X}^{\mu X} (\xi_{y_1,y_2})^{1/c-1} dy_1 dy_2 \,,$$

where $\xi_{y_1,y_2} \asymp X^c$. Consequently $B(X) \gg \vartheta X^{3-c}$. The lemma is proved. $\square$

3 Outline of the proof.

Consider the sum

$$\Gamma = \sum_{\substack{X<p_1,p_2,p_3\leq 2X \\ |p_1^c+p_2^c+p_3^c-N|<\vartheta \\ (p_i+2,P(z))=1,\, i=1,2,3}} \log p_1 \log p_2 \log p_3 \,. \tag{9}$$

Arguing as in [3] we obtain

$$\Gamma \geq 3\Gamma_1 - 2\Gamma_2 \,, \tag{10}$$

where

$$\Gamma_1 = \int\limits_{-\infty}^{\infty} \Upsilon(t) e(-Nt) L_1(t,X) L_2^2(t,X) dt \,,$$

$$\Gamma_2 = \int\limits_{-\infty}^{\infty} \Upsilon(t) e(-Nt) L_2^3(t,X) dt \,,$$

$$L_1(t,X) = \sum_{d|P(z)} \lambda^-(d) \sum_{\substack{X<p\leq 2X \\ p+2\equiv 0\,(d)}} e(tp^c) \log p \,, \tag{11}$$

$$L_2(t,X) = \sum_{d|P(z)} \lambda^+(d) \sum_{\substack{X<p\leq 2X \\ p+2\equiv 0\,(d)}} e(tp^c) \log p \,. \tag{12}$$

We shall consider the sum Γ_1. The sum Γ_2 can be considered in the same way. We divide Γ_1 into three parts

$$\Gamma_1 = \Gamma_1^{(1)} + \Gamma_1^{(2)} + \Gamma_1^{(3)} \,, \tag{13}$$

where

$$\Gamma_1^{(1)} = \int\limits_{|t|<\tau} \Upsilon(t)e(-Nt)L_1(t,X)L_2^2(t,X)dt\,, \tag{14}$$

$$\Gamma_1^{(2)} = \int\limits_{\tau\leq|t|\leq K} \Upsilon(t)e(-Nt)L_1(t,X)L_2^2(t,X)dt\,, \tag{15}$$

$$\Gamma_1^{(3)} = \int\limits_{|t|>K} \Upsilon(t)e(-Nt)L_1(t,X)L_2^2(t,X)dt\,. \tag{16}$$

We shall estimate $\Gamma_1^{(3)}$, $\Gamma_1^{(1)}$, $\Gamma_1^{(2)}$ respectively in the sections 4, 5, 6. In section 7 we shall complete the proof of the Theorem.

4 Upper bound for $\mathbf{\Gamma_1^{(3)}}$.

In the trivial region we state a single lemma.

Lemma 3 *For the integral $\Gamma_1^{(3)}$, defined by (16), we have*

$$\Gamma_1^{(3)} \ll 1\,. \tag{17}$$

We leave the proof to the reader.

5 Asymptotic formula for $\mathbf{\Gamma_1^{(1)}}$.

Let

$$G^{\pm} = \sum_{d|P(z)} \frac{\lambda^{\pm}(d)}{\varphi(d)}\,. \tag{18}$$

Following the method in [3] we find

$$\Gamma_1^{(1)} = B(X)G^{-}(G^{+})^2 + \mathcal{O}\left(\vartheta\frac{X^{3-c}}{\log^4 X}\right), \tag{19}$$

where

$$B(X) = \int\limits_X^{2X}\int\limits_X^{2X}\int\limits_X^{2X} \upsilon(y_1^c + y_2^c + y_3^c - N)dy_1dy_2dy_3\,.$$

According to Lemma 2 we have

$$B(X) \gg \vartheta X^{3-c}\,. \tag{20}$$

6 Upper bound for $\mathbf{\Gamma_1^{(2)}}$.

Arguing as in ([7], Lemma 7) we obtain

Lemma 4 *For the sums denoted by (11) and (12) we have*

$$\int\limits_0^1 |L_j(t,X)|^2 dt \ll X^{2-c}\log^5 X\,, \quad j=1,2\,.$$

Working similar to ([6], Lemma 1) and ([7], Lemma 8) we get

Lemma 5 *Assume that $\tau \le |\alpha| \le K$. Let $\xi(d)$ be complex number defined for $d \le D$, and let*

$$\xi(d) \ll 1\,. \tag{21}$$

Then for the sum

$$L(\alpha,X)=\sum_{d\le D}\xi(d)\sum_{\substack{X<p\le 2X\\ p+2\equiv 0\,(d)}} e(\alpha p^c)\log p \tag{22}$$

we have

$$L(\alpha,X) \ll \frac{X}{(\log X)^{A+10}}\,.$$

We next treat $\Gamma_1^{(2)}$, defined by (15). We have

$$\Gamma_1^{(2)} \ll \max_{\tau\le t\le K}|L_1(t,X)|\int\limits_\tau^K |\Upsilon(t)||L_2(t,X)|^2 dt\,. \tag{23}$$

Using Lemma 1 we find

$$\int\limits_\tau^K |\Upsilon(t)||L_2(t,X)|^2 dt \ll \vartheta\int\limits_\tau^{1/\vartheta}|L_2(t,X)|^2 dt+\int\limits_{1/\vartheta}^K |L_2(t,X)|^2\frac{dt}{t}$$

$$\ll \vartheta\sum_{0\le n\le 1/\vartheta}\int\limits_n^{n+1}|L_2(t,X)|^2 dt+\sum_{1/\vartheta-1\le n\le K}\frac{1}{n}\int\limits_n^{n+1}|L_2(t,X)|^2 dt\,.$$

The last estimate, (3), (5) and Lemma 4 give us

$$\int\limits_\tau^K |\Upsilon(t)||L_2(t,X)|^2 dt \ll X^{2-c}\log^6 X\,. \tag{24}$$

Therefore by (3), (23), (24) and Lemma 5 we obtain

$$\Gamma_1^{(2)} \ll \vartheta \frac{X^{3-c}}{\log^4 X} . \tag{25}$$

Summarizing (13), (17), (19) and (25) we get

$$\Gamma_1 = B(X) G^-(G^+)^2 + \mathcal{O}\left(\vartheta \frac{X^{3-c}}{\log^4 X}\right) . \tag{26}$$

7 Proof of the Theorem.

Since Γ_2 is estimated in the same way then from (10) and (26) we find

$$\Gamma \geq B(X) W + \mathcal{O}\left(\vartheta \frac{X^{3-c}}{\log^4 X}\right) , \tag{27}$$

where

$$W = 3 \left(G^+\right)^2 \left(G^- - \frac{2}{3} G^+\right) \tag{28}$$

and $G^{\pm}$ are defined by (18).
We put

$$\mathcal{F}(z) = \prod_{2<p\leq z} \left(1 - \frac{1}{p-1}\right) , \quad s = \frac{\log D}{\log z} . \tag{29}$$

Let $f(s)$ and $F(s)$ are the lower and the upper functions of the linear sieve. Arguing as in [3] we obtain

$$W \geq 3\mathcal{F}^3(z) \left(f(s) - \frac{2}{3} F(s) + \mathcal{O}\big((\log X)^{-1/3}\big)\right) . \tag{30}$$

Hence using (27) and (30) we get

$$\Gamma \geq 3B\mathcal{F}^3(z) \left(f(s) - \frac{2}{3} F(s) + \mathcal{O}\big((\log X)^{-1/3}\big)\right) + \mathcal{O}\left(\vartheta \frac{X^{3-c}}{\log^4 X}\right) . \tag{31}$$

For $2 \leq s \leq 3$ we have

$$f(s) = \frac{2e^{\gamma} \log(s-1)}{s} , \qquad F(s) = \frac{2e^{\gamma}}{s}$$

(γ denotes Euler's constant). We choose

$$s = 2.95.$$

Then by (6), (7) and (29) we find

$$\beta = 0.0968523\,.$$

It is not difficult to compute that for sufficiently large X we have

$$f(s) - \frac{2}{3}F(s) > 10^{-5}\,. \tag{32}$$

It remains to notice that

$$\mathcal{F}(z) \asymp \frac{1}{\log X}\,. \tag{33}$$

Therefore, using (6), (20), (31) – (33) we obtain

$$\Gamma \gg \vartheta \frac{X^{3-c}}{\log^3 X}\,. \tag{34}$$

From (3) and (34) it follows that $\Gamma \to \infty$ as $X \to \infty$.

Bearing in mind (3), (9) and (34) we conclude that for some constant $c_0 > 0$ there are at least $c_0 X^{3-c}(\log X)^{-A-6}$ triples of primes p_1, p_2, p_3 satisfying $X < p_1, p_2, p_3 \leq 2X$, $|p_1^c + p_2^c + p_3^c - N| < \vartheta$ and such that for any prime factor p of $p_j + 2$, $j = 1, 2, 3$ we have $p \geq X^{0.0968523}$.

The proof of the Theorem is complete.

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Received: October 30, 2017 **Reviewer:** Prof. PhD Ivan Trendafilov

Годишник на Технически Университет - София, том 67, книга 3, 2017
Proceedings of the Technical University of Sofia, Volume 67, Issue 3, 2017

ЙОРДАНОВИ ДИФЕРЕНЦИРАНИЯ В ПРЪСТЕНИ И ПОЛУПРЪСТЕНИ

Стоян Димитров

Резюме: *В тази статия е направен кратък обзор на Йордановите диференцирания в пръстени и полупръстени.*

Ключови думи: *Диференциална алгебра, Йорданово диференциране, пръстени, полупръстени.*

JORDAN DERIVATIONS ON RINGS AND SEMIRINGS

Stoyan Dimitrov

Abstract: *This paper presents a brief survey of the current state of the Jordan derivations on rings and semirings.*

Keywords: *Differential algebra, Jordan derivations, rings, semirings.*

1 Introduction

Differential algebra is an area of algebra in the study of algebraic structures equipped with finitely many derivations, that are linear functions satisfied Leibniz product rule. The notion of the ring with derivation (i.e. with differentiation) is old and plays a significant role in the integration of analysis, algebraic geometry and algebra – [33]. One of the most remarkable map is a Jordan derivation.

2 Preliminaries and historical note

Throughout the discussion, unless otherwise mentioned, R denotes an associative ring having at least two elements and S denotes a semiring – see [6] and [20]. The ring R (semiring S) may not have unity.

ISSN 1311-0829

Recall that a ring R is said to be *prime* if the product of any two nonzero ideals of R is nonzero. Equivalently $aRb = \{0\}$ with $a, b \in R$ implies $a = 0$ or $b = 0$. A ring R is called *semiprime* if it has no nonzero nilpotent ideals. Equivalently, $aRa = \{0\}$ with $a \in R$ implies $a = 0$.

A map $d : R \to R$ ($d : S \to S$) is a derivation of a ring R (semiring S) if d is additive and satisfies the Leibniz rule: $d(ab) = d(a)b + ad(b)$, for all $a, b \in R$ ($a, b \in S$).

In 1934 a new part of algebra called theory of Jordan algebras was initiated by M. Born, P. Jordan and E. Wigner (see [26]) in order to formalize the notion of an algebra of observables in quantum mechanics.

First fundamental boosk for Jordan algebras are [24] and [48].

In 1950's, Herstein initiated the study of the relationship between the associative and the Jordan and Lie structure of associative rings. Herstein constructed, starting from the ring R, a new ring, called Jordan ring R, defining the product in this one as being $a \circ b = ab + ba$ for any $a, b \in R$. This new product is well-defined and it can be easily verified that $(R, +, \circ)$ is a ring. Now, an additive mapping d, from the Jordan ring into itself, is said by Herstein to be a Jordan derivation, if $d(a \circ b) = d(a) \circ b + a \circ d(b)$, for every $a, b \in R$. Every derivation is obviously a Jordan derivation and the converse is in general not true.

In the year 1957, Herstein proved a classical result in this direction which becomes a jumping point for many workers later:

Theorem 1 ([23], Theorem 3.1) *Let R be a prime ring of characteristic different from 2, then every Jordan derivation of R is a derivation.*

3 Jordan derivation of prime rings

In 1988 Brešar and Vukman (see [13]) presented an alternative proof of Herstein result (Theorem 1). If we consider the proof of Theorem 1 we find that the assumption that the characteristic of R be different from 2 enters only in proving $d(aba) = d(a)ba + ad(b)a + abd(a)$ for every $a, b \in R$. So, if redefine a Jordan derivation by equalities $d(a^2) = d(a)a + ad(a)$ and and $d(aba) = d(a)ba + ad(b)a + abd(a)$, we can prove the following result:

Theorem 2 ([13], Theorem 3.4) *If R is a prime ring and d is a Jordan derivation (redefined) of R, then dis a derivation except if R is both commutative (and is an integral domain) and of characteristic 2.*

Later Brešar [8] proved that Herstein's result is true for 2-torsion free semiprime rings.

Brešar in [11] defined an additive mapping $d : R \to R$ to be a *Jordan triple derivation* if $d(aba) = d(a)ba + ad(b)a + abd(a)$ for every $a, b \in R$. He proved that every Jordan triple derivation of a 2-torsion-free semiprime ring is a derivation ([11], Theorem 4.3).

In 1984 Awtar extended the HersteinŸs theorem to Lie ideals – see citeAwtar.

Some of these results have been extended to different rings and algebras in various directions (see [9], [10], [12], [15], [32]).

In 2000 Ashraf and Rehman in [3] considered additive mappings $d : R \to R$ called a *Jordan left derivations* if $d(x^2) = 2xd(x)$ for all $x \in R$ and proved some results for Jordan left derivations in prime rings.

In 2014 T. Lee and J. Lin [28] proved the theorem

Theorem 3 ([28], Theorem 2.2) *Let R be a prime ring. An additive map $\delta : R \to Q_{ml}(R)$ is a Jordan derivation if and only if there exist a derivation $d : R \to Q_{ml}(R)$ and an additive map $\mu : R \to C$ such that $\delta = d + \mu$ and $\mu(x^2) = 0$ for all $x \in R$.*

4 Jordan derivations of matrix algebras

Recall Herstein's definition for Jordan homomorphisms from [22]:

Jordan homomorphism is a mapping $\varphi : R \to R'$ such that $\varphi(a + b) = \varphi(a) + \varphi(b)$ and $\varphi(ab + ba) = \varphi(a)\varphi(b) + \varphi(b)\varphi(a)$ for all $a, b \in R$.

By a classical result of Jacobson and Rickart [25] every full matrix ring over a 2-torsion free unital ring has no proper Jordan derivation. This fact can be obtained from the two following theorems, where A is a 2-torsion free unital ring and $M_n(A)$ is the ring of all $n \times n$ marices over A:

Theorem 4 ([25], Theorem 7) *Any Jordan homomorphism of $M_n(A)$ is the sum of a homomorphism and an anti-homomorphism.*

Theorem 5 ([25], Theorem 22) *If any Jordan homomorphism of A is the sum of a homomorphism and an anti-homomorphism, then every Jordan derivation from A into itself is a derivation.*

In 2009 Alizadeh in [2] proved the following theorem

Theorem 6 *Every Jordan derivation from $M_n(A)$ into $M_n(A)$ is a derivation.*

In 2007 N. Ghoseiri in [19] denote by R be a 2-torsionfree ring with identity and by S be a subring of the ring $M_n(A)$ that contains the ring $T_n(A)$ of all upper triangular matrices over R. The author proved that any Jordan derivation in S can be uniquely represented as the sum of a derivation and a special Jordan derivation.

In 2010 S. Zhao, J. Zhu in [47] considered so called Jordan all-derivable points in the algebra of all upper triangular matrices. The obtained theorems continued the earlier results in [49] and [50].

In 2013 Y. Li, L. Wyk, F. Wei in [31] construct some examples of Jordan derivations of generalized matrix algebras given by a Morita context.

In 2017 Y. Lee and C. Zheng [29] proved analogouss result for higher Jordan derivations.

5 Jordan derivation of triangular algebras

We recall the definition of triangular algebra.

Definition 1 *Triangular algebra $\mathcal{T} = Tri(\mathcal{A}, \mathcal{M}, \mathcal{B})$ is an algebra of the form*

$$Tri(\mathcal{A}, \mathcal{M}, \mathcal{B}) = \{ \begin{pmatrix} a & m \\ 0 & b \end{pmatrix} : a \in \mathcal{A}\,, m \in \mathcal{M}\,, b \in \mathcal{B} \}$$

under the usual matrix operations, where $\mathcal{A}$ and $\mathcal{B}$ are two algebras over a commutative ring $\mathcal{R}$, and $\mathcal{M}$ is an $(\mathcal{A}, \mathcal{B})$-bimodule which is faithful as a left $\mathcal{A}$-module and also as a right $\mathcal{B}$-module.

In 2005 Benkovič [7] determined Jordan derivations on triangular matrices over commutative rings and proved that every Jordan derivation from the algebra of all upper triangular matrices into its arbitrary unital bimodule is the sum of a derivation and an antiderivation.

In 2006 Zhang and Yu [45] showed that every Jordan derivation of triangular algebras is a derivation. More precisely, they proved the following result.

Theorem 7 *Let $\mathcal{A}, \mathcal{B}$ be unital algebras over a 2-torsion free commutative ring $\mathcal{R}$, and $\mathcal{M}$ be a unital $(\mathcal{A}, \mathcal{B})$-bimodule that is faithful as a left $\mathcal{A}$-module and also as a right $\mathcal{B}$-module. Then every Jordan derivation from the triangular algebra $Tri(\mathcal{A}, \mathcal{M}, \mathcal{B})$ into itself is a derivation.*

In 2007 Xuehan Cheng and Wu Jing [14] generalized the result of Zhang and Yu.

Theorem 8 *Let $\mathcal{A}$ and $\mathcal{B}$ be two algebras over a 2-torsion free commutative ring $\mathcal{R}$, with the property:*

(P) Suppose that $a \in \mathcal{A}\,(resp.\mathcal{B})$. If $xay + yax = 0$ holds for all $x, y \in \mathcal{A}\,(resp.\mathcal{B})$, then $a = 0$.

Let $\mathcal{M}$ be a faithful $(\mathcal{A}, \mathcal{B})$-bimodule and $\mathcal{T}$ be triangular algebra $Tri(\mathcal{A}, \mathcal{M}, \mathcal{B})$. Then every Jordan derivation δ on $\mathcal{T}$ into itself is a derivation.

In 2014 H. Ghahramani proved result about Jordan derivations on block upper triangular matrix algebras.

Theorem 9 *Let $\mathcal{T} = \mathcal{T}(n_1, n_2, ..., n_k)$ be a block upper triangular algebras in $M_n(\mathcal{C})$ $(n \geq 1)$ and $\mathcal{M}$ be a 2-torsion free unital $\mathcal{T}$-bimodule. Suppose that $D : \mathcal{T} \to \mathcal{M}$ is a Jordan derivation. Then there exist a derivation $d : \mathcal{T} \to \mathcal{M}$ and an antiderivation $\alpha : \mathcal{T} \to \mathcal{M}$ such that $D = d + \alpha$ and $\alpha(D(n_1, n_2, ..., n_k)) = \{0\}$. Moreover, d and α are uniquely determined.*

Similar results H. Ghahramani proved in [16].

For more information about Jordan derivation of triangular algebras we refer to ([1], [5], [18], [21], [27], [30] [44], [46]).

6 Jordan derivations in semirings

For semilattice $\mathcal{M}$ the set $\mathcal{E}_{\mathcal{M}}$ of the endomorphisms of $\mathcal{M}$ is a semiring with respect to the addition and multiplication defined with:

- $h = f + g$ when $h(x) = f(x) \vee g(x)$ for all $x \in \mathcal{M}$,
- $h = f \cdot g$ when $h(x) = f\left(g(x)\right)$ for all $x \in \mathcal{M}$.

This semiring is called the *endomorphism semirimg* of the semilattice $\mathcal{M}$.

In [34] – [37] and [39] – [42] all considered semirings are endomorphism semirings of a finite chain $\mathcal{C}_n = \left(\{0, 1, \ldots, n-1\}, \vee\right)$ denoted by $\mathcal{E}_{\mathcal{C}_n}$. This semiring can be considered as a simplex – [35]. For 1 – simplices, which are called *strings*, I. Trendafilov ([34]) construct Jordan derivations.

For 2 – simplices, which are called *triangles*, D. Vladeva ([39]) consider 10 types of Jordan derivations corresponding to 10 subsemirings of an arbitrary triangle constructed in [35]. In the same work [39] the author construct Jordan derivations in an arbitrary k–simplex an prove some facts for them and for so called local derivations. In [38] D. Vladeva consider derivations in upper triangular, Toeplitz and circulant matrices. Using the representations of the upper triangular Toeplitz $n \times n$ matrices with entries from an additively idempotent semiring from the last article, D. Vladeva in [39] construct a Jordan derivation which is not a derivation.

In [36] and [37] the authors considered a projections of the elements of a triangle on the three strings of a triangle and proved that these maps are derivations. These derivations are connected with Jordan derivations defined in [39]. So in [40], [41] and [42] D. Vladeva consider The Jordan derivations commuting with the projections on the smallest, middle and greatest strings of a triangle.

In [43] D. Vladeva find a plenty of examples of Jordan derivations in the semiring of upper triangular $n \times n$ matrices with entries from an additively idempotent semiring and prove that some of them form an additively idempotent semiring.

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Received: November 10, 2017 **Reviewer:** Prof. PhD Ivan Trendafilov

ИЗПОЛЗВАНЕ НА УНИВЕРСАЛНАТА МАТЕМАТИЧЕСКА СРЕДА "MATH-XPRESS" ЗА ПОДПОМАГАНЕ ПРЕПОДАВАНЕТО И ОЦЕНЯВАНЕТО В ЧАСОВЕТЕ ПО ЛИНЕЙНА АЛГЕБРА

Мариана Дурчева

Резюме: *В настоящата статия ние описваме основните характеристики на универсалната математическа среда "Math-XPress" и нейното използване за семинарна работа, за самостоятелно обучение на студентите, както и за оценяване на тяхната работа. Math-Xpress включва редактор на уравнения, 2D и 3D графичен плотер, оценител на изрази от областта на компютърната алгебра, както и постъпковото им решаване, възможности за прилагане на динамична геометрия (2D и 3D). В курса по Линейна алгебра със студенти от специалността «Приложна математика и информатика» сме използвали и генератора на задачи, който системата също позволява.*
Ключови думи: *Math-XPress, компютърна алгебра, линейна алгебра*

USING THE UNIVERSAL MATH ENVIRONMENT “MATH-XPRESS” FOR TEACHING AND ASSESSMENT OF LINEAR ALGEBRA

Mariana Durcheva

Abstract: *In the present work we describe the main features of the Universal Math Environment “Math-XPress” and its use for classroom teaching, home training and assessment of students. Math-Xpress includes linked modules of equation editor, 2D and 3D graph plotter, CAS expression evaluator and step-to-step solver, dynamic geometry (2D and 3D) and problem solving tutor. The Problem Generator has been used for the development of the course of Linear Algebra, for the group of students of the specialty „Applied Mathematics and Informatics”.*
Keywords*: Math-XPress, Computer Algebra System, Linear Algebra.*

1. INTRODUCTION

Math-Xpress includes a number of linked modules that can be used by Math teachers to improve the quality of lectures and the overall effectiveness of class and home work [1]. Among those are: *Equation editor*, *2D* and *3D graph plotter*, *CAS Expression Evaluator and step-to-step solver*, *Dynamic geometry (2D and 3D)* and *Problem solving Tutor*.

XPress-Editor is graphical formula editor, enabling natural WYSIWYG editing of math expressions, that does not require familiarity with a special syntax. To enable that, *XPress-Editor* uses Editor Keys, that represent Math templates and special symbols. The resulting expression can be either pasted into Word or other text editors, or

ISSN 1311-0829

used by *XPress-graph plotter*, CAS based *XPress-evaluator* or *XPress-Tutor*.
XPress-graph plotter enables plotting and exploring graphs of functions of 2 and 3 variables (2D and 3D), families of functions and intersections of several graphs or graphs and figures (in plane) or solids (in space).
XPress-evaluator performs operations on algebraic expressions. The subjects covered by *XPress-evaluator* include: Arithmetic, Elementary Algebra, Trigonometry, Calculus, Probability and Statistics, Linear Algebra, Complex numbers. *XPress-evaluator* shows the result either in final form or as a number of step-by-step operations; in the case when the equation does not have an algebraic solution, it is solved numerically.
Math-Xpress includes also two modules of interactive dynamic geometry:
2-D and 3-D XPress-geometry Explorer, which are in turn interrelated to other modules, enabling, for instance, to show the intersections of graphs of functions with figures and solids, or geometry representation of complex numbers, etc.
The objects created by *XPress-editor*, *Graph Plotter and Geometry Explorer* can be imbedded into Word or pdf-pages and called from them directly in interactive Math-Xpress environment.
As long as the above mentioned modules offer tools rather than content, they do not depend on specific curriculum, nor on the language of teaching. On the contrary, the main objective of *XPress-Tutor* is Training and Assessment of the students learning at native language the courses according to a local curriculum. Language settings of Math-XPress allow to choose the required one (Fig. 1), and to get the tasks according to the curriculum and language of the country.

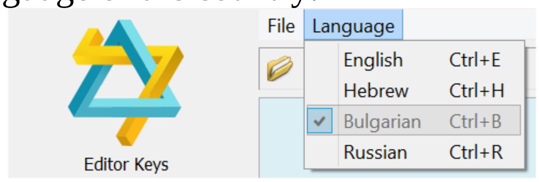


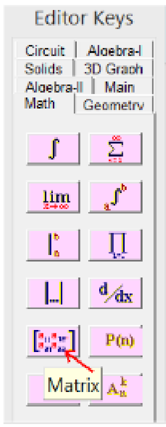
Fig. 1.

2. USING "MATH-XPRESS" BY A TEACHER DURING THE SEMINARS FOR INTERACTIVE DEMONSTRATIONS OF BASIC OPERATIONS IN LINEAR ALGEBRA

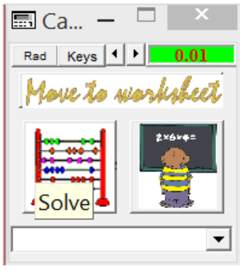
XPress-evaluator enables to present many subjects on Math in live and attractive form, by demonstrating, when possible, graphical representation of functions, equations, inequalities, complex numbers, vectors, etc., or detailed operations on Math objects, showing their dependence on the constants and parameters that define them.
In Linear Algebra such operations include:

a) *Calculation of determinants*

XPress-evaluator calculates determinants of and order up to 3x3 in detailed form. For doing that, open the folder Math, click on the button Matrix, and fill in the template in Edit window, for instance, matrix of the second order:



Click on the Solve button and obtain the detailed calculation in the Work Sheet window:



WorkSheet

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = a\cdot d-b\cdot c = a\cdot d-b\cdot c$$

The elements of the matrix can be also expressions, like:

WorkSheet

$$\begin{vmatrix} a-5 & b-x \\ c-\sqrt{x} & d+\frac{1}{2\cdot x+x^3} \end{vmatrix} = (a-5)\cdot\left(d+\frac{1}{2\cdot x+x^3}\right)-(b-x)\cdot\left(c-\sqrt{x}\right) = (a-5)\cdot d+\frac{a-5}{2\cdot x+x^3}-b\cdot\left(c-\sqrt{x}\right)+\left(c-\sqrt{x}\right)\cdot x$$

The calculation is also in detailed form, when a determinant is of higher order, as in the following example:

$$\begin{vmatrix} t+2 & -1 & 1 \\ 3 & t-2 & 1 \\ 4 & -4 & t+5 \end{vmatrix} = (t+2)\cdot\begin{vmatrix} t-2 & 1 \\ -4 & t+5 \end{vmatrix} - 3\cdot\begin{vmatrix} -1 & 1 \\ -4 & t+5 \end{vmatrix} + 4\cdot\begin{vmatrix} -1 & 1 \\ t-2 & 1 \end{vmatrix} = \left(t^2-10+3\cdot t\right)\cdot(t+2)+4\cdot(t+2)-t+7$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & m \end{vmatrix} = a\cdot\begin{vmatrix} e & f \\ h & m \end{vmatrix} - d\cdot\begin{vmatrix} b & c \\ h & m \end{vmatrix} + g\cdot\begin{vmatrix} b & c \\ e & f \end{vmatrix} = e\cdot a\cdot m-a\cdot f\cdot h-b\cdot d\cdot m+c\cdot d\cdot h+b\cdot f\cdot g-e\cdot c\cdot g$$

The teacher can demonstrate during the seminar the solution of the problem which is one of the test exercises (yet with another coefficients), the ability of an interactive changing of the elements adds essentially to the understanding of the solution:

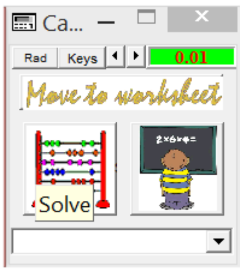
$$\begin{vmatrix} t+3 & -2 & 2 \\ 4 & t-3 & 2 \\ 5 & -5 & t+7 \end{vmatrix} = (t+3)\cdot\begin{vmatrix} t-3 & 2 \\ -5 & t+7 \end{vmatrix} - 4\cdot\begin{vmatrix} -2 & 2 \\ -5 & t+7 \end{vmatrix} + 5\cdot\begin{vmatrix} -2 & 2 \\ t-3 & 2 \end{vmatrix} = \left(t^2-21+4\cdot t\right)\cdot(t+3)+10\cdot(t+3)-2\cdot t+26$$

b) *Operations with matrices*

XPress-evaluator performs and shows in details all the basic operations with matrices, like:

1. Addition of matrices: $(a_{ij}) + (b_{ij}) = (a_{ij} + b_{ij})$

Edit the exercise in Edit Window and click on Solve button:



$$\begin{bmatrix} 4 & -5 \\ 0 & 1 \\ -6 & 1 \end{bmatrix} + \begin{bmatrix} 3 & 2 \\ 7 & -4 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -3 \\ 7 & -3 \\ -8 & 2 \end{bmatrix}$$

2. Multiplication of a real number c and an m x n matrix $A = (a_{ij})$: $cA = (ca_{ij})$:

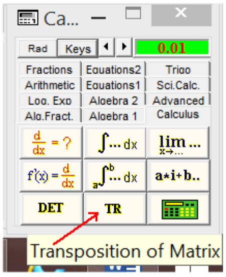
$$3\cdot\begin{bmatrix} 4 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 12 & -3 \\ 6 & 9 \end{bmatrix}$$

3. Product of two matrices $A = (a_{ij})$ and $B = (b_{ij})$:

$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{in}b_{nj}$:

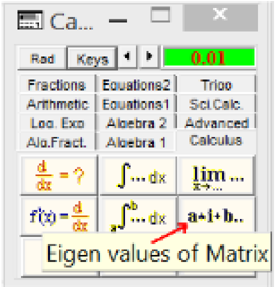
$$\begin{bmatrix} 1 & 2 & -3 \\ 4 & 0 & -2 \end{bmatrix} \cdot \begin{bmatrix} 5 & -4 & 2 & 0 \\ -1 & 6 & 3 & 1 \\ 7 & 0 & 4 & 8 \end{bmatrix} = \begin{bmatrix} -18 & 8 & -4 & -22 \\ 6 & -16 & 0 & -16 \end{bmatrix}$$

4. Transposition of a matrix: $A = (a_{ij}) \Rightarrow A' = (a'_{ij}) = (a_{ji})$:



$$\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\7 & 8 & 9\end{bmatrix}^T = \begin{bmatrix}1 & 4 & 7\\2 & 5 & 8\\3 & 6 & 9\end{bmatrix}$$

5. Eigenvalues of a matrix 2x2:



$\begin{bmatrix}2 & 3\\4 & 5\end{bmatrix}$ The initial matrix

$\begin{vmatrix}2\text{-}t & 3\\4 & 5\text{-}t\end{vmatrix} = t^2 - 7\cdot t - 2 = 0$ The secular equation

$t_1 = 7.27;$

$t_2 = -0.27$

The eigen values

Calcluation of eigen vectors

$t = 7.27$:

$\begin{cases}3\cdot x_2 - 5.27\cdot x_1 = 0\\4\cdot x_1 - 2.27\cdot x_2 = 0\end{cases}$ system accepted

$\begin{cases}-5.27\cdot x_1 + 3\cdot x_2 = 0\\0 = 0\end{cases}$

➔

$\begin{cases}x_1 = 0.57\cdot x_2\\x_2 = \text{any number}\end{cases}$

Dim W=1

$(x_2 = 1)$, $(0.57,\ 1)$ Roots are found

$t = -0.27$:

$\begin{cases}3\cdot x_2 + 2.27\cdot x_1 = 0\\4\cdot x_1 + 5.27\cdot x_2 = 0\end{cases}$ system accepted

$\begin{cases}2.27\cdot x_1 + 3\cdot x_2 = 0\\0 = 0\end{cases}$

$\begin{cases}x_1 = -1.32\cdot x_2\\x_2 = \text{any number}\end{cases}$

Dim W=1

$(x_2 = 1)$, $(-1.32,\ 1)$ Roots are found

6. Eigenvalues of a matrix 3x3:

$\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\7 & 8 & 9\end{bmatrix}$ The initial matrix

$\begin{vmatrix}1\text{-}t & 2 & 3\\4 & 5\text{-}t & 6\\7 & 8 & 9\text{-}t\end{vmatrix} = -t^3 + 15\cdot t^2 + 18\cdot t = 0$ The secular equation

$t_1 = 0;$

$t_2 = 16.12;$

$t_3 = -1.12$

The eigen values

Calcluation of eigen vectors

$t = 0.00$:

$\begin{cases}x_1 + 2\cdot x_2 + 3\cdot x_3 = 0\\4\cdot x_1 + 5\cdot x_2 + 6\cdot x_3 = 0\\7\cdot x_1 + 8\cdot x_2 + 9\cdot x_3 = 0\end{cases}$ system accepted

Dim W=1

$(x_3 = 1)$, $(0.28,\ 0.64,\ 1)$ Roots are found

$t = -1.12$:

$\begin{cases}2\cdot x_2 + 3\cdot x_3 + 2.12\cdot x_1 = 0\\4\cdot x_1 + 6\cdot x_3 + 6.12\cdot x_2 = 0\\7\cdot x_1 + 8\cdot x_2 + 10.12\cdot x_3 = 0\end{cases}$ system accepted

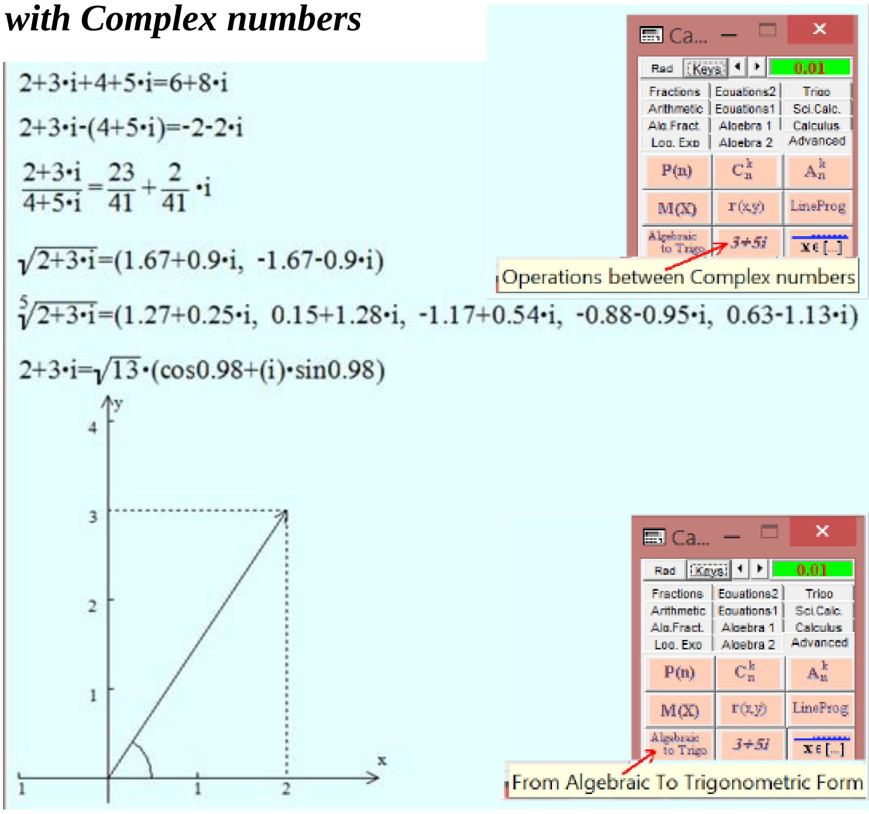
Dim W=1

$(x_3 = 1)$, $(1,\ -2,\ 1)$ Roots are found

$t = 16.12$:

$\begin{cases}2\cdot x_2 + 3\cdot x_3 - 15.12\cdot x_1 = 0\\4\cdot x_1 + 6\cdot x_3 - 11.12\cdot x_2 = 0\\7\cdot x_1 + 8\cdot x_2 - 7.12\cdot x_3 = 0\end{cases}$ system accepted

c) *Operations with Complex numbers*



d) *Linear operators*

This topic can also be developed using the system that generates tasks with different parameters:

Задача 5.21 Нека T е линеен оператор от R^2 в R^2, дефиниран по следния начин:

$$T(1,\ 2)=[2\ \ -1],\ T(2,\ 1)=[4\ \ 1]$$

Да се намери матрицата на линейния оператор $T(a,\ b)$.

Задача 5.22 Нека T е линеен оператор от R^2 към R^3, дефиниран по следния начин:

$$T(-1,\ 0)=[-1\ \ 0\ \ -2],\ T(1,\ 1)=[0\ \ 2\ \ 3]$$

Намерете матрицата на линейния оператор $T(a,\ b)$.

Задача 5.23 Нека T е линеен оператор от R^3 към R^3, дефиниран чрез формулата:

$$T(x,\ y,\ z)=[2x-y+2z\ \ x+y\ \ x-2y+2z]$$

Да се намери новата база на линейния оператор.

Задача 5.24 Нека T е линеен оператор от R^3 към R^4, дефиниран с помощта на формулата:

$$T(x,\ y,\ z)=[x+y-2z\ \ -2y+z\ \ x-y-z\ \ 2x-3z]$$

Да се намери размерността на новополучената система вектори.

Задача 5.25 Нека T е линеен оператор от R^3 към R^3, дефиниран чрез формулата:

$$T(x,\ y,\ z)=[2x-y+2z\ \ x+y\ \ x-2y+2z]$$

Да се намери базата на ядрото на линейния оператор.

Задача 5.26 Нека T е линеен оператор от R^3 към R^4, дефиниран чрез формулата:

$$T(x,\ y,\ z,\ t)=[2x-y+z+2t\ \ x+2y-z-t\ \ x-3y+z+2t]$$

Намерете размерността на ядрото на линейния оператор.

Задача 5.27 Намерете линеен оператор T от R^3 към R^4, ако е известно, че векторите в новото пространство имат следните координати:

$$[2\ \ -1\ \ 0\ \ 1] \text{ и } [1\ \ 2\ \ 1\ \ -1].$$

Задача 5.28 Намерете линеен оператор T от R^4 към R^3, ако е известно, че ядрото на линейния оператор се състои от векторите:

$$[-1\ \ 2\ \ 0\ \ 1] \text{ и } [1\ \ 0\ \ -1\ \ 1].$$

Задача 5.29 Линейният оператор T от R^3 към R^3 е дефиниран чрез формулата:

$$T(x,\ y,\ z) = [-x+2y\ \ \ 2x+y+2z\ \ \ -2x-2y+2z]$$

Намерете матрицата на оператора T.

Задача 5.30 Линейният оператор T от R^3 към R^3 е дефиниран чрез формулата:

$$T(x,\ y,\ z) = [x-2y+z\ \ \ x+3y+5z\ \ \ -2x-y+5z]$$

Намерете матрицата на оператора T в базата S:

$$S = \{[-1\ \ 2\ \ 4],\ [-2\ \ -2\ \ -3],\ [2\ \ 1\ \ 1]\}$$

3. USING MATH-XPRESS" FOR TRAINING OF STUDENTS AND ASSESSMENT FOR THE COURSE IN LINEAR ALGEBRA

XPress-Tutor offers a student the series of problems, organized into weekly tasks, according to the course curriculum [2]. For the course in Linear Algebra taught at the Technical University of Sofia, the following tasks have been developed:
Systems of linear equations, Basics of Determinants, Basics of Matrix, Vector's Spaces-1, Vector's Spaces-2, Vector's Spaces-3, Vector's Spaces-4, Complex Numbers, Determinants – Advanced problems, Matrix – Advanced problems - altogether 100 exercises, divided onto 10 weekly tasks. Each task is presented in three modes of operation: **Learning, Training** and **Test** (Fig 2).

Fig. 2.

During the Learning mode, a student is offered a series of problems on a given subject; each problem includes random parameters, so that different runs exhibit different initial sets of the parameters.
A student can solve the problem in his way, by entering an answer using the Editor

Keys. The program checks an input expression and responds either by question mark, if the answer does not coincide with any possible answer, or by "Wrong" remark, if it coincides with one of the predicted wrong answers (usually typical) (Fig. 3).

Fig. 3.

A student can also ask for a Help that is presented in 3 levels:

1) General Help, where a method of solution that is common to all the problems of a specific subject is described (Fig 4);

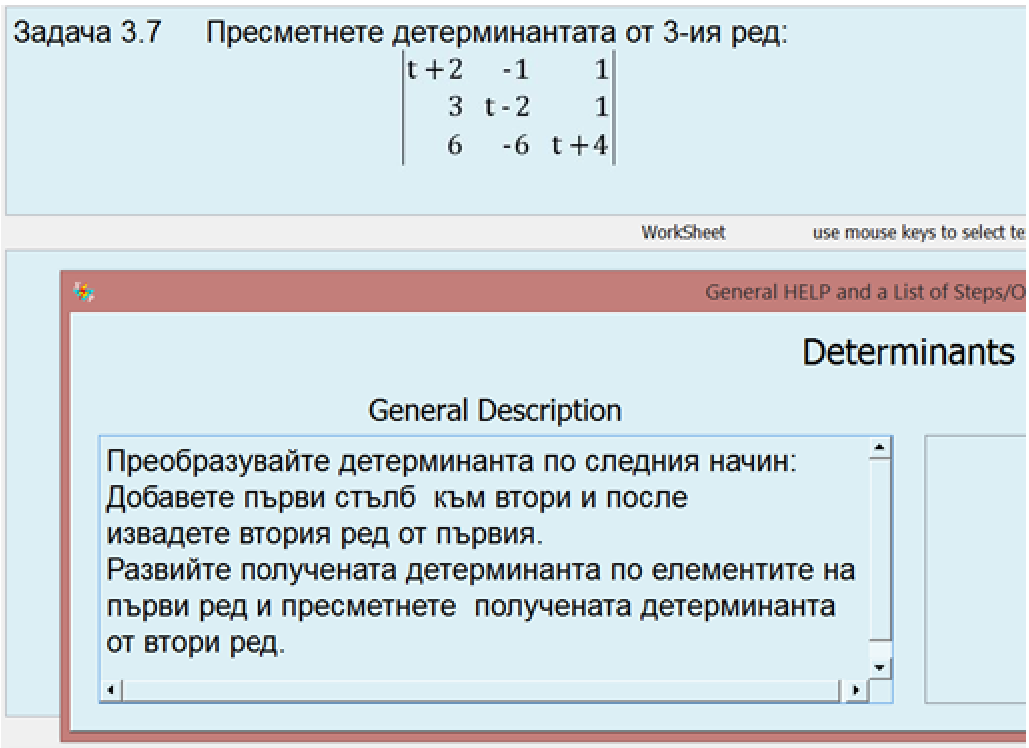


Fig. 4.

2) A List of Steps of a problem's solution and the description of each step (Fig 5, 6 and 7):

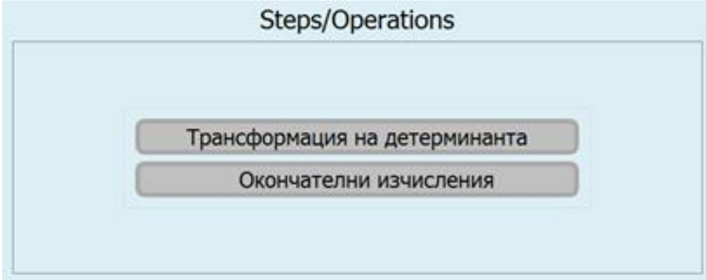


Fig. 5.

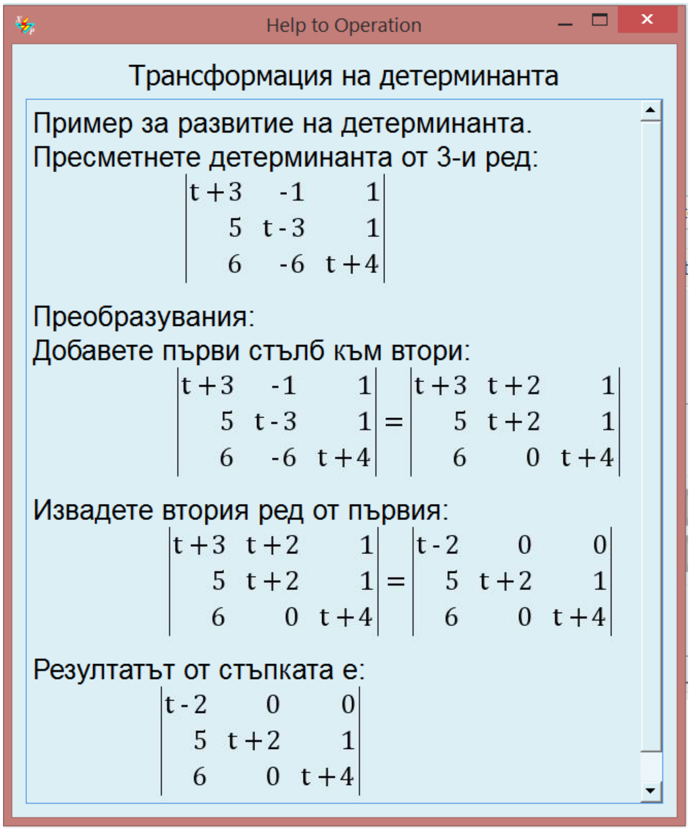


Fig. 6.

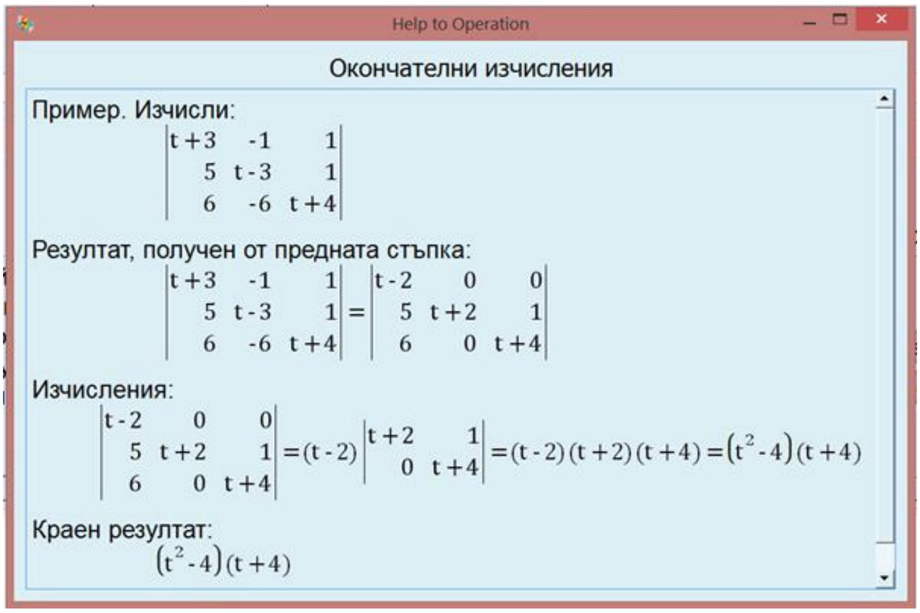


Fig. 7.

At any moment the student can enter his/her proposal for the next step of the solution, and the program will compare it with possible answers (both correct and typical wrong ones), and respond correspondingly.
If, however, he/she while being in **Learning** mode, still couldn't suggest the correct result of one of the following steps (or the final answer), clicking on **Hint** button, he/she will see the correct result of the nearest unsolved step.
In **Training** mode, the student obtains 4 possible answers (one true and 3 wrong), and in order to proceed, he/she has to find the correct one (Fig. 8); however a number of trials in training mode is unlimited.

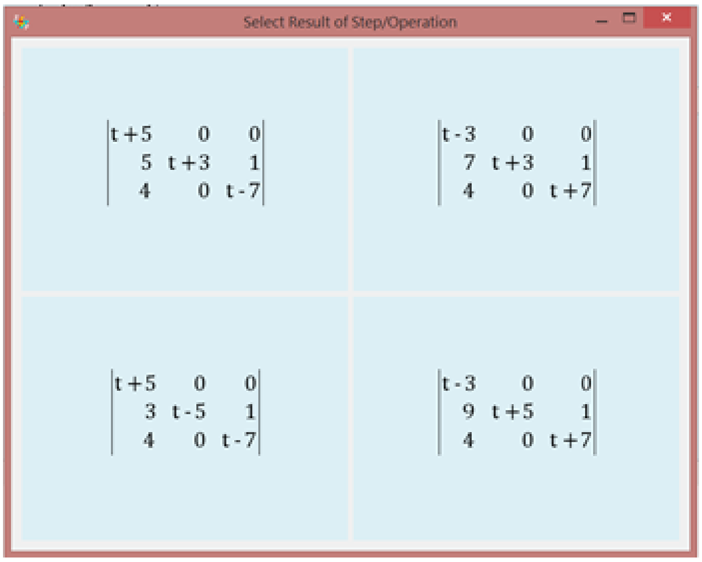


Fig. 8.

In **Test** mode, Help and Hint are not available, and a student has only one attempt to find the correct answer.

In Learning and Training modes, after finishing all the steps of the solution, the student can either move to the next problem, or repeat the current one with a new set of initial data.

In both Learning and Training modes **all the features of Math Xpress are available**, so that in a course of problem solving the student can explore the problem using different tools, like *XPress-graph plotter* or *XPress-evaluator* that can help him/her in better understanding of a solution.

In typical student's activity scenario, he/she usually starts with Test mode, looking at the set of the problems and trying to realize whether he/she can solve them without help.

The system allows to exit a test, without reducing the marks. However, this is allowed only twice for each problem, otherwise the mark for that problem will be null.
While being out of the Test, the student can enter Learning or Training modes, and learns how to solve the problem, similar to that in which he/she has encountered difficulties: the difference between the problems presented in Test and other modes is usually in the values of the random parameters, defining the problem.

4. DEVELOPMENT OF NEW CONTENT

The problems are developed using the *XPress Problem Generator* – external module, enabling compiling of new items by unexperienced in programming people [3].

During the current academic year a course of Linear Algebra has been taught using the Universal Math Environment “Math-XPress” to a group of 28 students of the Technical University of Sofia, and its use has indicated an improved interest of the students to the subject, as well as an essential improvement of the tests results.

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Received: November 09, 2017 **Reviewer:** Prof. PhD Ivan Trendafilov

Годишник на Технически Университет - София, том 67, книга 3, 2017
Proceedings of the Technical University of Sofia, Volume 67, Issue 3, 2017

ЙОРДАНОВИ ДИФЕРЕНЦИРАНИЯ, КОМУТИРАЩИ С ПРОЕКЦИЯТА ВЪРХУ НАЙ-МАЛКИЯ СТРИНГ НА ТРИЪГЪЛНИК

Димитринка Владева

Резюме: *Целта на тази статия е да се намерят Йордановите диференцирания, които комутират с проекцията ∂_1 на $\triangle^{(n)}\{a, b, c\}$ върху $\mathcal{STR}^{(n)}\{a, b\}$.*

Ключови думи: *полупръстен от ендоморфизми на крайна верига, диференциална алгебра, Йорданови диференцирания, диференцирания в полупръстени.*

JORDAN DERIVATIONS COMMUTING WITH THE PROJECTION ON THE SMALLEST STRING OF A TRIANGLE

Dimitrinka Vladeva

Abstract: *The aim of this paper is to find the Jordan derivations commuting with projection ∂_1 of $\triangle^{(n)}\{a, b, c\}$ on $\mathcal{STR}^{(n)}\{a, b\}$.*

Keywords: *endomorphism semiring of a finite chain, differential algebra, Jordan derivations, derivations in semirings.*

1 Introduction and preliminaries

Since this article and ***Jordan derivation commuting with the projection on the greatest string of a triangle***, published in the same volume, are closely related, here we use the references of this paper.

Jordan algebras and Jordan derivations were introduced in 1934 by P. Jordan [6] to formalize the notion of an algebra of observables in quantum mechanics.

In 1957 Herstein [5] proved that every Jordan derivation from a prime ring of characteristic different from two into itself is a derivation. This result has been extended to different rings and algebras in various directions (see [1], [2], [3] and [7]).

About derivations in semirings we use

• the definition in Golan's book [4],

• the projections on the strings of a triangle, which are derivations – [9] and [10],

• the projections on the strings of an arbitrary endomorphism semiring, which are derivations – [12].

ISSN 1311-0829

An algebra $R = (R, +, .)$ with two binary operations $+$ and $\cdot$ on R, is called a *semiring* if:

1. $(R, +)$ is a commutative semigroup,

2. $(R, \cdot)$ is a semigroup,

3. distributive laws hold $x \cdot (y + z) = x \cdot y + x \cdot z$ and $(x + y) \cdot z = x \cdot z + y \cdot z$ for any $x, y, z \in R$.

For a join-semilattice $(\mathcal{M}, \vee)$ set $\mathcal{E}_{\mathcal{M}}$ of the endomorphisms of $\mathcal{M}$ is a semiring with respect to the addition and multiplication defined by:

• $h = f + g$ when $h(x) = f(x) \vee g(x)$ for all $x \in \mathcal{M}$,

• $h = f \cdot g$ when $h(x) = f\left(g(x)\right)$ for all $x \in \mathcal{M}$.

This semiring is called the *endomorphism semiring* of $\mathcal{M}$.

In this article, all semilattices are finite chains. We fix a finite chain $\mathcal{C}_n = (\{0, 1, \ldots, n-1\}, \vee)$ and denote the endomorphism semiring of this chain by $\widehat{\mathcal{E}}_{\mathcal{C}_n}$. We do not assume that $\alpha(0) = 0$ for arbitrary $\alpha \in \widehat{\mathcal{E}}_{\mathcal{C}_n}$. So, there is not a zero in endomorphism semiring $\widehat{\mathcal{E}}_{\mathcal{C}_n}$.

Here we give a new treatment of the subsemirings of endomorphism semiring $\widehat{\mathcal{E}}_{\mathcal{C}_n}$ of a finite chain. For arbitrary elements $a_0, a_1, \ldots, a_{k-1} \in \mathrm{C}_n$, where $k \leq n$ and $a_0 < a_1 < \ldots < a_{k-1}$ we denote $A = \{a_0, a_1, \ldots, a_{k-1}\}$. Now, consider endomorphisms $\alpha \in \widehat{\mathcal{E}}_{\mathcal{C}_n}$ with $Im(\alpha) \subseteq A$. The set of the all such endomorphisms α is a maximal simplex. We denote this simplex by $\sigma_k^{(n)}(A) = \sigma^{(n)}\{a_0, a_1, \ldots, a_{k-1}\}$. The endomorphisms $\alpha \in \sigma^{(n)}\{a_0, a_1, \ldots, a_{k-1}\}$ such that

$$\alpha(0) = \cdots = \alpha(i_0 - 1) = a_0, \alpha(i_0) = \cdots = \alpha(i_0 + i_1 - 1) = a_1, \cdots$$

$$\alpha(i_0 + \cdots + i_{k-2}) = \cdots = \alpha(i_0 + \cdots + i_{k-1} - 1) = a_{k-1}$$

we denote by $\alpha = (a_0)_{i_0}(a_1)_{i_1} \ldots (a_{k-1})_{i_{k-1}}$, where $\displaystyle\sum_{p=0}^{k-1} i_p = n$.

Particularly in case $n = 2$ the simplex is callecd a string and is denoted by $\mathcal{STR}^{(n)}\{a, b\}$ and in case $n = 3$ the simplex is called a triangle and is denoted by $\triangle^{(n)}\{a, b, c\}$ - see [8]. For Jordan derivations in arbitrary endomorphism semiring the reader is refer to [11].

2 Jordan derivations commuting with derivation ∂_1

The projection on the smallest string of the triangle

$$\partial_1 : \triangle^{(n)}\{a, b, c\} \to \mathcal{STR}^{(n)}\{a, b\}$$

such that for any $\alpha = a_i b_j c_{n-i-j}$,

$$\partial_1(\alpha) = a_i b_{n-i} \in \mathcal{STR}^{(n)}\{a, b\}$$

is a derivation as we proved in Theorem 1 of [9]. The maximal subsemiring $\mathcal{TRAP}\{\overline{a}, \overline{b}, b_{b+1}c_{n-b-1}, a_{b+1}c_{n-b-1}\}$ of the triangle closed under the derivation ∂_1 is consisting of all types except (a, c, c), (b, c, c) and (c, c, c) – Theorem 2 of [9].

Let $\beta = a_k b_\ell c_{n-k-\ell}$, where $a \leq k-1 < b \leq k+\ell-1 < c$ is an element of $\mathcal{RI}\left(\triangle^{(n)}\{a, b, c\}\right)$. Then for any $\alpha \in \mathcal{TRAP}\{\overline{a}, \overline{b}, b_{b+1}c_{n-b-1}, a_{b+1}c_{n-b-1}\}$, it follows that Jordan multiplication

$$\partial_\beta(\alpha) = \alpha\beta + \beta\alpha = \alpha + \beta\alpha$$

is a derivation – see [11].

Theorem 1. *Let* $\alpha \in \mathcal{TRAP}\{\overline{a}, \overline{b}, b_{b+1}c_{n-b-1}, a_{b+1}c_{n-b-1}\}$ *and* $\beta \in \mathcal{RI}\left(\triangle^{(n)}\{a, b, c\}\right)$. *Then* $\partial_1\partial_\beta(\alpha) = \partial_\beta\partial_1(\alpha)$.

Proof. Case 1. Let $\alpha \in (a, a, a)$. Then $\beta\alpha = \overline{a}$ and $\partial_\beta(\alpha) = \alpha$. Since $\partial_1(\alpha) \in (a, a, a)$, it follows $\partial_1\partial_\beta(\alpha) = \partial_\beta(\partial_1(\alpha)) = \partial_1(\alpha) = \partial_1(\partial_\beta(\alpha)) = \partial_\beta\partial_1(\alpha)$.

Case 2. Let $\alpha \in (a, a, b)$. Then $\alpha = a_i b_j c_{n-i-j}$, where $a < b \leq i-1 < c \leq i+j-1$. We obtain $\beta\alpha = a_k b_\ell c_{n-k-\ell} \cdot a_i b_j c_{n-i-j} = a_{k+\ell} b_{n-k-\ell}$ and then $\partial_\beta(\alpha) = \alpha + a_{k+\ell} b_{n-k-\ell}$. Since $\partial_1(\alpha) = a_i b_{n-i} \in (a, a, b)$, it follows $\beta\partial_1(\alpha) = a_k b_\ell c_{n-k-\ell} \cdot a_i b_{n-i} = a_{k+\ell} b_{n-k-\ell}$. Thus $\partial_1\partial_\beta(\alpha) = \partial_\beta(\partial_1(\alpha)) = \partial_1(\alpha) + \beta\partial_1(\alpha) = \partial_1(\alpha) + a_{k+\ell} b_{n-k-\ell}$. Hence,

$$\partial_\beta\partial_1(\alpha) = \partial_1(\partial_\beta(\alpha)) = \partial_1(\alpha + a_{k+\ell} b_{n-k-\ell}) = \partial_1(\alpha) + a_{k+\ell} b_{n-k-\ell} = \partial_1\partial_\beta(\alpha).$$

Case 3. Let $\alpha \in (a, b, b)$. Then $\alpha = a_i b_j c_{n-i-j}$, where $a \leq i-1 < b < c \leq i+j-1$. We obtain $\beta\alpha = a_k b_\ell c_{n-k-\ell} \cdot a_i b_j c_{n-i-j} = a_k b_{n-k}$ and then $\partial_\beta(\alpha) = \alpha + a_k b_{n-k}$. Since $\partial_1(\alpha) = a_i b_{n-i} \in (a, b, b)$, it follows $\beta\partial_1(\alpha) = a_k b_\ell c_{n-k-\ell} \cdot a_i b_{n-i} = a_k b_{n-k}$. Thus, we have $\partial_1\partial_\beta(\alpha) = \partial_\beta(\partial_1(\alpha)) = \partial_1(\alpha) + \beta\partial_1(\alpha) = \partial_1(\alpha) + a_k b_{n-k}$. Hence,

$$\partial_\beta\partial_1(\alpha) = \partial_1(\partial_\beta(\alpha)) = \partial_1(\alpha + a_k b_{n-k}) = \partial_1(\alpha) + a_k b_{n-k} = \partial_1\partial_\beta(\alpha).$$

Case 4. Let $\alpha \in (b, b, b)$. Then $\alpha = a_i b_j c_{n-i-j}$, where $i \leq a < b < c \leq i+j-1$. We obtain $\beta\alpha = \overline{b}$ and then $\partial_\beta(\alpha) = \alpha + \overline{b} = \alpha$. Since $\partial_1(\alpha) \in (b, b, b)$, it follows

$$\partial_1\partial_\beta(\alpha) = \partial_\beta(\partial_1(\alpha)) = \partial_1(\alpha) = \partial_1(\partial_\beta(\alpha)) = \partial_\beta\partial_1(\alpha).$$

Case 5. Let $\alpha \in (a, a, c)$. Then $\alpha = a_i b_j c_{n-i-j}$, where $a < b \leq i-1 < i+j \leq c$. We obtain $\beta\alpha = a_k b_\ell c_{n-k-\ell} \cdot a_i b_j c_{n-i-j} = a_{k+\ell} c_{n-k-\ell}$ and then $\partial_\beta(\alpha) = \alpha + a_{k+\ell} c_{n-k-\ell}$. Since $\partial_1(\alpha) = a_i b_{n-i}$, it follows $\beta\partial_1(\alpha) = a_k b_\ell c_{n-k-\ell} \cdot a_i b_{n-i} = a_{k+\ell} b_{n-k-\ell}$. Thus $\partial_1\partial_\beta(\alpha) = \partial_\beta(\partial_1(\alpha)) = \partial_1(\alpha) + \beta\partial_1(\alpha) = \partial_1(\alpha) + a_{k+\ell} b_{n-k-\ell}$. Hence,

$$\partial_\beta\partial_1(\alpha) = \partial_1(\partial_\beta(\alpha)) = \partial_1(\alpha + a_{k+\ell} c_{n-k-\ell}) = \partial_1(\alpha) + a_{k+\ell} b_{n-k-\ell} = \partial_1\partial_\beta(\alpha).$$

Case 6. Let $\alpha \in (a, b, c)$. Then $\alpha = a_i b_j c_{n-i-j}$, where $a \leq i-1 < b \leq i+j-1 < c$. Then α is a right identity and $\partial_\beta(\alpha) = \alpha + \beta$. So,

$$\partial_\beta \partial_1(\alpha) = \partial_1(\partial_\beta(\alpha)) = \partial_1(\alpha + \beta) = \partial_1(\alpha) + \partial_1(\beta) = \partial_1(\alpha) + a_k b_{n-k}.$$

Since $\partial_1(\alpha) = a_i b_{n-i}$, it follows $\beta\partial_1(\alpha) = a_k b_\ell c_{n-k-\ell} \cdot a_i b_{n-i} = a_k b_{n-k}$. Hence,

$$\partial_1 \partial_\beta(\alpha) = \partial_\beta(\partial_1(\alpha)) = \partial_1(\alpha) + \beta\partial_1(\alpha) = \partial_1(\alpha) + a_k b_{n-k} = \partial_\beta \partial_1(\alpha).$$

Case 7. Let $\alpha \in (b, b, c)$. Then $\alpha = a_i b_j c_{n-i-j}$, where $i \leq a < b < i + j \leq c$. We obtain $\beta\alpha = a_k b_\ell c_{n-k-\ell} \cdot a_i b_j c_{n-i-j} = b_{k+\ell} c_{n-k-\ell}$ and then $\partial_\beta(\alpha) = \alpha + b_{k+\ell} c_{n-k-\ell}$. Since $\partial_1(\alpha) = a_i b_{n-i}$, it follows $\beta\partial_1(\alpha) = a_k b_\ell c_{n-k-\ell} \cdot a_i b_{n-i} = \overline{b}$. Thus, we have

$$\partial_1 \partial_\beta(\alpha) = \partial_\beta(\partial_1(\alpha)) = \partial_1(\alpha) + \beta\partial_1(\alpha) = \partial_1(\alpha) + \overline{b}.$$

Hence, $\partial_\beta \partial_1(\alpha) = \partial_1(\partial_\beta(\alpha)) = \partial_1(\alpha + b_{k+\ell} c_{n-k-\ell}) = \partial_1(\alpha) + \overline{b} = \partial_1 \partial_\beta(\alpha)$.

Let $\beta \in (a, a, a)$. Then for any α, it follows $\partial_\beta(\alpha) = \alpha\beta + \beta\alpha = \overline{a} + \beta\alpha = \beta\alpha$.

Theorem 2. *Let $\alpha \in \mathcal{TRAP}\{\overline{a}, \overline{b}, b_{b+1} c_{n-b-1}, a_{b+1} c_{n-b-1}\}$ and $\beta \in (a, a, a)$. Then $\partial_1 \partial_\beta(\alpha) = \partial_\beta \partial_1(\alpha)$.*

Proof. Case 1. Let $\alpha(a) = a$. Then $\beta\alpha = \overline{a}$ and $\partial_\beta(\alpha) = \overline{a}$. Now $\partial_\beta \partial_1(\alpha) = \partial_1(\partial_\beta(\alpha)) = \partial_1(\overline{a}) = \overline{a}$. Since $\partial_1(\alpha)(a) = a$, it follows

$$\partial_1 \partial_\beta(\alpha) = \partial_\beta(\partial_1(\alpha)) = \partial_\beta(\overline{a}) = \overline{a} = \partial_\beta \partial_1(\alpha).$$

Case 2. Let $\alpha(a) = b$. Then $\beta\alpha = \overline{b}$ and $\partial_\beta(\alpha) = \overline{b}$. Now $\partial_\beta \partial_1(\alpha) = \partial_1(\partial_\beta(\alpha)) = \partial_1(\overline{b}) = \overline{b}$. Since $\partial_1(\alpha)(a) = b$, it follows $\partial_1 \partial_\beta(\alpha) = \partial_\beta(\partial_1(\alpha)) = \partial_\beta(\overline{b}) = \overline{b} = \partial_\beta \partial_1(\alpha)$.

Let $\beta \in (b, b, b)$. Then for any α, it follows $\partial_\beta(\alpha) = \alpha\beta + \beta\alpha = \overline{b} + \beta\alpha$.

Theorem 3. *Let $\alpha \in \mathcal{TRAP}\{\overline{a}, \overline{b}, b_{b+1} c_{n-b-1}, a_{b+1} c_{n-b-1}\}$ and $\beta \in (b, b, b)$. Then $\partial_1 \partial_\beta(\alpha) = \partial_\beta \partial_1(\alpha)$.*

Proof. Case 1. Let $\alpha(b) = a$. Then $\beta\alpha = \overline{a}$ and $\partial_\beta(\alpha) = \overline{b}$. Now $\partial_\beta \partial_1(\alpha) = \partial_1(\partial_\beta(\alpha)) = \partial_1(\overline{b}) = \overline{b}$. Since $\partial_1(\alpha)(b) = a$, it follows

$$\partial_1 \partial_\beta(\alpha) = \partial_\beta(\partial_1(\alpha)) = \partial_\beta(\overline{b}) = \overline{b} = \partial_\beta \partial_1(\alpha).$$

Case 2. Let $\alpha(b) = b$. Then $\beta\alpha = \overline{b}$ and $\partial_\beta(\alpha) = \overline{b}$. Now $\partial_\beta \partial_1(\alpha) = \partial_1(\partial_\beta(\alpha)) = \partial_1(\overline{b}) = \overline{b}$. Since $\partial_1(\alpha)(b) = b$, it follows $\partial_1 \partial_\beta(\alpha) = \partial_\beta(\partial_1(\alpha)) = \partial_\beta(\overline{b}) = \overline{b} = \partial_\beta \partial_1(\alpha)$.

Not all Jordan derivations commutes with the derivation ∂_1 as we now show.

Proposition 1. *Let $\alpha \in (a, a, a)$ or $\alpha \in (a, a, c)$ and $\beta \in (a, a, b)$. Then, it follows $\partial_1 \partial_\beta(\alpha) \neq \partial_\beta \partial_1(\alpha)$.*

Proof. Let $\beta = a_k b_\ell c_{n-k-\ell}$, where $a < b \leq k-1 < c \leq k+\ell-1$.

Case 1. Let $\alpha = a_i b_j c_{n-i-j}$, where $a < b \leq i-1 < i+j \leq c$. We obtain $\partial_\beta(\alpha) =$

$$= a_i b_j c_{n-i-j} \cdot a_k b_\ell c_{n-k-\ell} + a_k b_\ell c_{n-k-\ell} \cdot a_i b_j c_{n-i-j} = a_{i+j} b_{n-i-j} + \overline{a} = a_{i+j} b_{n-i-j}.$$

Then $\partial_\beta \partial_1(\alpha) = \partial_1(\partial_\beta(\alpha)) = \partial_1(a_{i+j} b_{n-i-j}) = a_{i+j} b_{n-i-j}$.

Since $\partial_1(\alpha) = a_i b_{n-i}$, it follows $\partial_1 \partial_\beta(\alpha) = \partial_\beta(\partial_1(\alpha)) = \partial_\beta(a_i b_{n-i}) =$

$$= a_i b_{n-i} \cdot a_k b_\ell c_{n-k-\ell} + a_k b_\ell c_{n-k-\ell} \cdot a_i b_{n-i} = \overline{a} + \overline{a} = \overline{a} < a_{i+j} b_{n-i-j} = \partial_\beta \partial_1(\alpha).$$

Case 2. Let $\alpha = a_i b_j c_{n-i-j}$, where $a < b \leq i-1 < i+j \leq c$. We obtain

$$\partial_\beta(\alpha) = a_i b_j c_{n-i-j} \cdot a_k b_\ell c_{n-k-\ell} + a_k b_\ell c_{n-k-\ell} \cdot a_i b_j c_{n-i-j} = a_{i+j} b_{n-i-j} + a_{k+\ell} c_{n-k-\ell}.$$

Then $\partial_\beta \partial_1(\alpha) = \partial_1(\partial_\beta(\alpha)) = \partial_1(a_{i+j} b_{n-i-j} + a_{k+\ell} c_{n-k-\ell}) = a_{i+j} b_{n-i-j} + a_{k+\ell} b_{n-k-\ell}$.

But $i+j \leq c < k+\ell$. Thus $a_{i+j} b_{n-i-j} > a_{k+\ell} b_{n-k-\ell}$ and then $\partial_\beta \partial_1(\alpha) = a_{i+j} b_{n-i-j}$. Hence $\partial_1 \partial_\beta(\alpha) = \partial_\beta(\partial_1(\alpha)) = \partial_\beta(a_i b_{n-i}) = a_i b_{n-i} \cdot a_k b_\ell c_{n-k-\ell} + a_k b_\ell c_{n-k-\ell} \cdot a_i b_{n-i} = \overline{a} + a_{k+\ell} b_{n-k-\ell} = a_{k+\ell} b_{n-k-\ell} < a_{i+j} b_{n-i-j} = \partial_\beta \partial_1(\alpha)$.

As in Proposition 1 we can prove that if $\alpha \in (a, a, b)$ or $\alpha \in (a, b, b)$, or $\alpha \in (a, b, c)$ and $\beta \in (a, a, b)$ then it follows $\partial_1 \partial_\beta(\alpha) \neq \partial_\beta \partial_1(\alpha)$.

Theorem 4. *Let $\alpha \in (b, b, b)$ or $\alpha \in (b, b, c)$ and $\beta \in (a, a, b)$. Then $\partial_1 \partial_\beta(\alpha) = \partial_\beta \partial_1(\alpha)$.*

Proof. Let $\beta = a_k b_\ell c_{n-k-\ell}$, where $a < b \leq k-1 < c \leq k+\ell-1$.

Case 1. Let $\alpha \in (b, b, b)$. Then $\alpha = a_i b_j c_{n-i-j}$, where $i \leq a < b < c \leq i+j-1$. We obtain $\beta\alpha = \overline{b}$ and $\alpha\beta = a_{i+j} b_{n-i-j}$. Then $\partial_\beta(\alpha) = \alpha\beta + \beta\alpha = a_{i+j} b_{n-i-j} + \overline{b} = \overline{b}$. Thus $\partial_\beta \partial_1(\alpha) = \partial_1(\partial_\beta(\alpha)) = \partial_1(\overline{b}) = \overline{b}$.

Since $\partial_1(\alpha) = a_i b_{n-i} \in (b, b, b)$, it follows $\beta \partial_1(\alpha) = \overline{b}$ and $\partial_1(\alpha)\beta = \overline{a}$. Hence

$$\partial_1 \partial_\beta(\alpha) = \partial_\beta(\partial_1(\alpha)) = \beta \partial_1(\alpha) + \partial_1(\alpha)\beta = \overline{b} + \overline{a} = \overline{b} = \partial_\beta \partial_1(\alpha).$$

Case 2. Let $\alpha \in (b, b, c)$. Then $\alpha = a_i b_j c_{n-i-j}$, where $i \leq a < b \leq i+j-1 < c$.

Now we obtain $\beta\alpha = b_{k+\ell} c_{n-k-\ell}$ and $\alpha\beta = a_{i+j} b_{n-i-j}$. Then $\partial_\beta(\alpha) = \alpha\beta + \beta\alpha = b_{k+\ell} c_{n-k-\ell} + a_{i+j} b_{n-i-j}$. Thus

$$\partial_\beta \partial_1(\alpha) = \partial_1(\partial_\beta(\alpha)) = \partial_1(b_{k+\ell} c_{n-k-\ell} + a_{i+j} b_{n-i-j}) = \overline{b} + a_{i+j} b_{n-i-j} = \overline{b}.$$

Since $\partial_1(\alpha) = a_i b_{n-i} \in (b, b, b)$, it follows $\beta \partial_1(\alpha) = \overline{b}$ and $\partial_1(\alpha)\beta = \overline{a}$. Hence

$$\partial_1 \partial_\beta(\alpha) = \partial_\beta(\partial_1(\alpha)) = \beta \partial_1(\alpha) + \partial_1(\alpha)\beta = \overline{b} + \overline{a} = \overline{b} = \partial_\beta \partial_1(\alpha).$$

Theorem 5. *Let $\alpha \in \mathcal{TRAP}\{\overline{a}, \overline{b}, b_{b+1} c_{n-b-1}, a_{b+1} c_{n-b-1}\}$ and $\beta \in (a, b, b)$. Then $\partial_1 \partial_\beta(\alpha) = \partial_\beta \partial_1(\alpha)$.*

Proof. Let $\beta = a_k b_\ell c_{n-k-\ell}$, where $a \leq k-1 < b < c \leq k+\ell-1$.

Case 1. Let $\alpha \in (a,a,a)$. Now $\alpha = a_i b_j c_{n-i-j}$, $a < b < c \leq i-1 < c$. Then

$$\partial_\beta(\alpha) = \alpha\beta + \beta\alpha = a_i b_j c_{n-i-j} \cdot a_k b_\ell c_{n-k-\ell} + a_k b_\ell c_{n-k-\ell} \cdot a_i b_j c_{n-i-j} = a_i b_{n-i} + \overline{a} = a_i b_{n-i}.$$

We obtain $\partial_\beta \partial_1(\alpha) = \partial_1(\partial_\beta(\alpha)) = \partial_1(a_i b_{n-i}) = a_i b_{n-i}$. Since $\partial_1(\alpha) = a_i b_{n-i}$, it follows $\beta\partial_1(\alpha) = a_k b_\ell c_{n-k-\ell} \cdot a_i b_{n-i} = \overline{a}$ and $\partial_1(\alpha)\beta = a_i b_{n-i} \cdot a_k b_\ell c_{n-k-\ell} = a_i b_{n-i}$. Hence $\partial_1 \partial_\beta(\alpha) = \partial_\beta(\partial_1(\alpha)) = \beta\partial_1(\alpha) + \partial_1(\alpha)\beta = \overline{a} + a_i b_{n-i} = a_i b_{n-i} = \partial_\beta \partial_1(\alpha)$.

Case 2. Let $\alpha \in (a,a,b)$. Now $\alpha = a_i b_j c_{n-i-j}$, where $a < b \leq i-1 < c \leq i+j-1$ and then $\partial_\beta(\alpha) = \alpha\beta + \beta\alpha = a_i b_j c_{n-i-j} \cdot a_k b_\ell c_{n-k-\ell} + a_k b_\ell c_{n-k-\ell} \cdot a_i b_j c_{n-i-j} = a_i b_{n-i} + a_{k+\ell} b_{n-k-\ell}$. Since $i \leq c \leq k+\ell$ we find $a_{k+\ell} b_{n-k-\ell} \leq a_i b_{n-i}$ and $\partial_\beta(\alpha) = a_i b_{n-i}$. Thus $\partial_\beta \partial_1(\alpha) = \partial_1(\partial_\beta(\alpha)) = \partial_1(a_i b_{n-i}) = a_i b_{n-i}$.

As in the previous case since $\partial_1(\alpha) = a_i b_{n-i}$, it follows $\beta\partial_1(\alpha) = a_k b_\ell c_{n-k-\ell} \cdot a_i b_{n-i} = a_{k+\ell} b_{n-k-\ell}$ and $\partial_1(\alpha)\beta = a_i b_{n-i} \cdot a_k b_\ell c_{n-k-\ell} = a_i b_{n-i}$. Hence

$$\partial_1 \partial_\beta(\alpha) = \partial_\beta(\partial_1(\alpha)) = \beta\partial_1(\alpha) + \partial_1(\alpha)\beta = a_{k+\ell} b_{n-k-\ell} + a_i b_{n-i}.$$

But $i \leq c < k+\ell$, so $a_{k+\ell} b_{n-k-\ell} < a_i b_{n-i}$. Thus $\partial_1 \partial_\beta(\alpha) = a_i b_{n-i} = \partial_\beta \partial_1(\alpha)$.

Case 3. Let $\alpha \in (a,b,b)$. Now $\alpha = a_i b_j c_{n-i-j}$, where $a \leq i-1 < b < c \leq i+j-1$ and then $\partial_\beta(\alpha) = \alpha\beta + \beta\alpha = a_i b_j c_{n-i-j} \cdot a_k b_\ell c_{n-k-\ell} + a_k b_\ell c_{n-k-\ell} \cdot a_i b_j c_{n-i-j} = a_i b_{n-i} + a_k b_{n-k}$.

Now we find $\partial_\beta \partial_1(\alpha) = \partial_1(\partial_\beta(\alpha)) = \partial_1(a_i b_{n-i} + a_k b_{n-k}) = a_i b_{n-i} + a_k b_{n-k}$.

As in the previous two cases since $\partial_1(\alpha) = a_i b_{n-i}$, it follows

$$\partial_1 \partial_\beta(\alpha) = \partial_\beta(\partial_1(\alpha)) = \beta\partial_1(\alpha) + \partial_1(\alpha)\beta =$$

$$= a_k b_\ell c_{n-k-\ell} \cdot a_i b_{n-i} + a_i b_{n-i} \cdot a_k b_\ell c_{n-k-\ell} = a_k b_{n-k} + a_i b_{n-i} = \partial_\beta \partial_1(\alpha).$$

Case 4. Let $\alpha \in (b,b,b)$. Then $\alpha\beta = \overline{b}$, $\beta\alpha = \overline{b}$ and $\partial_\beta(\alpha) = \overline{b}$. We obtain

$$\partial_\beta \partial_1(\alpha) = \partial_1(\partial_\beta(\alpha)) = \partial_1(\overline{b}) = \overline{b}.$$

Since $\partial_1(\alpha) \in (b,b,b)$, it follows $\beta\partial_1(\alpha) = \overline{b}$ and $\partial_1(\alpha)\beta = \overline{b}$. Hence

$$\partial_1 \partial_\beta(\alpha) = \partial_\beta(\partial_1(\alpha)) = \beta\partial_1(\alpha) + \partial_1(\alpha)\beta = \overline{b} + \overline{b} = \overline{b} = \partial_\beta \partial_1(\alpha).$$

Case 5. Let $\alpha \in (a,b,c)$. Then $\alpha = a_i b_j c_{n-i-j}$, where $a < i \leq b < i+j \leq c$. Now α is a right identity and $\partial_\beta(\alpha) = \alpha\beta + \beta$. But $\alpha\beta = a_i b_j c_{n-i-j} \cdot a_k b_\ell c_{n-k-\ell} = a_i b_{n-i}$. So, $\partial_\beta(\alpha) = a_i b_{n-i} + a_k b_\ell c_{n-k-\ell}$.

We obtain $\partial_\beta \partial_1(\alpha) = \partial_1(\partial_\beta(\alpha)) = \partial_1(a_i b_{n-i} + a_k b_\ell c_{n-k-\ell}) = a_i b_{n-i} + a_k b_{n-k}$.

Since $\partial_1(\alpha) = a_i b_{n-i}$, it follows $\partial_1 \partial_\beta(\alpha) = \partial_\beta(\partial_1(\alpha)) =$

$$= \beta\partial_1(\alpha) + \partial_1(\alpha)\beta = a_k b_\ell c_{n-k-\ell} \cdot a_i b_{n-i} + a_i b_{n-i} \cdot a_k b_\ell c_{n-k-\ell} = a_k b_{n-k} + a_i b_{n-i} = \partial_\beta \partial_1(\alpha).$$

Case 6. Let $\alpha \in (b,b,c)$. Then $\alpha = a_i b_j c_{n-i-j}$, where $i \leq a < b \leq i+j-1 < c$. We obtain $\partial_\beta(\alpha) = \alpha\beta + \beta\alpha = a_i b_{n-i} + b_{k+\ell} c_{n-k-\ell}$. Thus

$$\partial_\beta \partial_1(\alpha) = \partial_1(\partial_\beta(\alpha)) = \partial_1(a_i b_{n-i} + b_{k+\ell} c_{n-k-\ell}) = a_i b_{n-i} + \overline{b} = \overline{b}.$$

Since $\partial_1(\alpha) = a_i b_{n-i}$, it follows $\partial_1\partial_\beta(\alpha) = \partial_\beta(\partial_1(\alpha)) = \partial_1(\alpha)\beta + \beta\partial_1(\alpha) =$

$$= a_i b_{n-i} \cdot a_k b_\ell c_{n-k-\ell} + a_k b_\ell c_{n-k-\ell} \cdot a_i b_{n-i} = \overline{b} + a_i b_{n-i} = \overline{b} = \partial_\beta\partial_1(\alpha).$$

Case 7. Let $\alpha \in (a, a, c)$. Then $\alpha = a_i b_j c_{n-i-j}$, where $a < b \leq i-1 < c \leq i+j-1$. We obtain $\partial_\beta(\alpha) = \alpha\beta + \beta\alpha = a_i b_j c_{n-i-j} \cdot a_k b_\ell c_{n-k-\ell} + a_k b_\ell c_{n-k-\ell} \cdot a_i b_j c_{n-i-j} = a_i b_{n-i} + a_{k+\ell} c_{n-k-\ell}$. Thus

$$\partial_\beta\partial_1(\alpha) = \partial_1(\partial_\beta(\alpha)) = \partial_1(a_i b_{n-i} + a_{k+\ell} c_{n-k-\ell}) = a_i b_{n-i} + a_{k+\ell} b_{n-k-\ell}.$$

But $i \leq c < k + \ell$ implies $a_{k+\ell} c_{n-k=\ell} < a_i b_{n-i}$. So $\partial_\beta\partial_1(\alpha) = a_i b_{n-i}$.

Since $\partial_1(\alpha) = a_i b_{n-i}$, it follows $\partial_1\partial_\beta(\alpha) = \partial_\beta(\partial_1(\alpha)) = \partial_1(\alpha)\beta + \beta\partial_1(\alpha) =$

$$= a_i b_{n-i} \cdot a_k b_\ell c_{n-k-\ell} + a_k b_\ell c_{n-k-\ell} \cdot a_i b_{n-i} = a_i b_{n-i} + a_{k+\ell} b_{n-k-\ell} = a_i b_{n-i} = \partial_\beta\partial_1(\alpha).$$

Theorem 6. *Let* $\alpha \in \mathcal{TRAP}\{\overline{a}, \overline{b}, b_{b+1} c_{n-b-1}, a_{b+1} c_{n-b-1}\}$ *and* $\beta \in (b, b, c)$. *Then* $\partial_1\partial_\beta(\alpha) = \partial_\beta\partial_1(\alpha)$.

Proof. Let $\beta = a_k b_\ell c_{n-k-\ell}$, where $k \leq a < b \leq k + \ell \leq c$.

Case 1. Let $\alpha \in (a, a, a)$. Now $\alpha = a_i b_j c_{n-i-j}$, where $a < b < c \leq i - 1$. We find

$$\partial_\beta(\alpha) = a_i b_j c_{n-i-j} \cdot a_k b_\ell c_{n-k-\ell} + a_k b_\ell c_{n-k-\ell} \cdot a_i b_j c_{n-i-j} = b_{i+j} c_{n-i-j} + \overline{a} = b_{i+j} c_{n-i-j}.$$

We obtain $\partial_\beta\partial_1(\alpha) = \partial_1(\partial_\beta(\alpha)) = \partial_1(b_{i+j} c_{n-i-j}) = \overline{b}$. Since $\partial_1(\alpha) = a_i b_{n-i}$, it follows $\beta\partial_1(\alpha) = a_k b_\ell c_{n-k-\ell} \cdot a_i b_{n-i} = \overline{a}$ and $\partial_1(\alpha)\beta = a_i b_{n-i} \cdot a_k b_\ell c_{n-k-\ell} = \overline{b}$. Hence $\partial_1\partial_\beta(\alpha) = \partial_\beta(\partial_1(\alpha)) = \beta\partial_1(\alpha) + \partial_1(\alpha)\beta = \overline{a} + \overline{b} = \overline{b} = \partial_\beta\partial_1(\alpha)$.

Case 2. Let $\alpha \in (a, a, b)$. Now $\alpha = a_i b_j c_{n-i-j}$, where $a < b \leq i - 1 < c$. Then

$$\partial_\beta(\alpha) = a_i b_j c_{n-i-j} \cdot a_k b_\ell c_{n-k-\ell} + a_k b_\ell c_{n-k-\ell} \cdot a_i b_j c_{n-i-j} = b_{i+j} c_{n-i-j} + a_{k+\ell} b_{n-k-\ell}.$$

Now we obtain $\partial_\beta\partial_1(\alpha) = \partial_1(\partial_\beta(\alpha)) = \partial_1(b_{i+j} c_{n-i-j} + a_{k+\ell} b_{n-k-\ell}) = \partial_1(b_{i+j} c_{n-i-j}) + \partial_1(a_{k+\ell} b_{n-k-\ell}) = \overline{b} + a_{k+\ell} b_{n-k-\ell} = \overline{b}$.

Since $\partial_1(\alpha) = a_i b_{n-i}$, it follows $\beta\partial_1(\alpha) = a_k b_\ell c_{n-k-\ell} \cdot a_i b_{n-i} = a_{k+\ell} b_{n-k-\ell}$ and $\partial_1(\alpha)\beta = a_i b_{n-i} \cdot a_k b_\ell c_{n-k-\ell} = \overline{b}$. Hence

$$\partial_1\partial_\beta(\alpha) = \partial_\beta(\partial_1(\alpha)) = \beta\partial_1(\alpha) + \partial_1(\alpha)\beta = a_{k+\ell} b_{n-k-\ell} + \overline{b} = \overline{b} = \partial_\beta\partial_1(\alpha).$$

Case 3. Let $\alpha \in (a, b, b)$. Now $\alpha = a_i b_j c_{n-i-j}$, where $a \leq i-1 < b < c \leq i+j-1$ and then $\partial_\beta(\alpha) = \alpha\beta + \beta\alpha = a_i b_j c_{n-i-j} \cdot a_k b_\ell c_{n-k-\ell} + a_k b_\ell c_{n-k-\ell} \cdot a_i b_j c_{n-i-j} = b_{i+j} c_{n-i-j} + a_k b_{n-k}$.

Now we find $\partial_\beta\partial_1(\alpha) = \partial_1(\partial_\beta(\alpha)) = \partial_1(b_{i+j} c_{n-i-j} + a_k b_{n-k}) = \overline{b} + a_k b_{n-k} = \overline{b}$.

Since $\partial_1(\alpha) = a_i b_{n-i}$, it follows $\partial_1\partial_\beta(\alpha) = \partial_\beta(\partial_1(\alpha)) = \beta\partial_1(\alpha) + \partial_1(\alpha)\beta = a_k b_\ell c_{n-k-\ell} \cdot a_i b_{n-i} + a_i b_{n-i} \cdot a_k b_\ell c_{n-k-\ell} = a_k b_{n-k} + \overline{b} = \overline{b} = \partial_\beta\partial_1(\alpha)$.

Case 4. Let $\alpha \in (b,b,b)$. As in case 4 of the previous theorem we find $\alpha\beta = \overline{b}$, $\beta\alpha = \overline{b}$ and $\partial_\beta(\alpha) = \overline{b}$. We obtain $\partial_\beta\partial_1(\alpha) = \partial_1(\partial_\beta(\alpha)) = \partial_1(\overline{b}) = \overline{b}$. Since $\partial_1(\alpha) \in (b,b,b)$, it follows $\beta\partial_1(\alpha) = \overline{b}$ and $\partial_1(\alpha)\beta = \overline{b}$. Hence

$$\partial_1\partial_\beta(\alpha) = \partial_\beta(\partial_1(\alpha)) = \beta\partial_1(\alpha) + \partial_1(\alpha)\beta = \overline{b} + \overline{b} = \overline{b} = \partial_\beta\partial_1(\alpha).$$

Case 5. Let $\alpha \in (a,b,c)$. Then $\alpha = a_i b_j c_{n-i-j}$, where $a < i \leq b < i+j \leq c$. Now α is a right identity and $\partial_\beta(\alpha) = \alpha\beta + \beta$. But $\alpha\beta = a_i b_j c_{n-i-j} \cdot a_k b_\ell c_{n-k-\ell} = b_{i+j} c_{n-i-j}$. So, $\partial_\beta(\alpha) = b_{i+j}c_{n-i-j} + a_k b_\ell c_{n-k-\ell}$.

We obtain $\partial_\beta\partial_1(\alpha) = \partial_1(\partial_\beta(\alpha)) = \partial_1(b_{i+j}c_{n-i-j} + a_k b_\ell c_{n-k-\ell}) =$

$$= \partial_1(b_{i+j}c_{n-i-j}) + \partial_1(a_k b_\ell c_{n-k-\ell}) = \overline{b} + a_k b_{n-k} = \overline{b}.$$

Since $\partial_1(\alpha) = a_i b_{n-i}$, it follows

$$\partial_1\partial_\beta(\alpha) = \partial_\beta(\partial_1(\alpha)) = \beta\partial_1(\alpha) + \partial_1(\alpha)\beta = a_k b_\ell c_{n-k-\ell} \cdot a_i b_{n-i} + a_i b_{n-i} \cdot a_k b_\ell c_{n-k-\ell} =$$

$$= a_k b_{n-k} + \overline{b} = \overline{b} = \partial_\beta\partial_1(\alpha).$$

Case 6. Let $\alpha \in (b,b,c)$. Then $\alpha = a_i b_j c_{n-i-j}$, where $i \leq a < b \leq i+j-1 < c$. We obtain $\partial_\beta(\alpha) = \alpha\beta + \beta\alpha = b_{i+j}c_{n-i-j} + b_{k+\ell}c_{n-k-\ell}$. Thus

$$\partial_\beta\partial_1(\alpha) = \partial_1(\partial_\beta(\alpha)) = \partial_1(b_{i+j}c_{n-i-j} + b_{k+\ell}c_{n-k-\ell}) =$$

$$= \partial_1(b_{i+j}c_{n-i-j}) + \partial_1(b_{k+\ell}c_{n-k-\ell}) = \overline{b} + \overline{b} = \overline{b}.$$

Since $\partial_1(\alpha) = a_i b_{n-i}$, it follows $\partial_1\partial_\beta(\alpha) = \partial_\beta(\partial_1(\alpha)) = \partial_1(\alpha)\beta + \beta\partial_1(\alpha) =$

$$= a_i b_{n-i} \cdot a_k b_\ell c_{n-k-\ell} + a_k b_\ell c_{n-k-\ell} \cdot a_i b_{n-i} = \overline{b} + \overline{b} = \overline{b} = \partial_\beta\partial_1(\alpha).$$

Case 7. Let $\alpha \in (a,a,c)$. Then $\alpha = a_i b_j c_{n-i-j}$, where $a < b \leq i-1 < c \leq i+j-1$. We obtain $\partial_\beta(\alpha) = \alpha\beta + \beta\alpha = a_i b_j c_{n-i-j} \cdot a_k b_\ell c_{n-k-\ell} + a_k b_\ell c_{n-k-\ell} \cdot a_i b_j c_{n-i-j} = b_{i+j}c_{n-i-j} + a_{k+\ell}c_{n-k-\ell}$.

Since $k+\ell \leq c < i+j$ we have $n-i-j < n-k-\ell$ and then $b_{i+j}c_{n-i-j} + a_{k+\ell}c_{n-k-\ell} = b_{k+\ell}c_{n-k-\ell}$.

Thus $\partial_\beta\partial_1(\alpha) = \partial_1(\partial_\beta(\alpha)) = \partial_1(b_{k+\ell}c_{n-k-\ell}) = \overline{b}$.

Since $\partial_1(\alpha) = a_i b_{n-i}$, it follows $\partial_1\partial_\beta(\alpha) = \partial_\beta(\partial_1(\alpha)) = \partial_1(\alpha)\beta + \beta\partial_1(\alpha) =$

$$= a_i b_{n-i} \cdot a_k b_\ell c_{n-k-\ell} + a_k b_\ell c_{n-k-\ell} \cdot a_i b_{n-i} = \overline{b} + a_{k+\ell}b_{n-k-\ell} = \overline{b} = \partial_\beta\partial_1(\alpha).$$

Proposition 2. *Let $\alpha \in (a,a,a)$ and $\beta \in (a,a,c)$. Then $\partial_1\partial_\beta(\alpha) \neq \partial_\beta\partial_1(\alpha)$.*

Proof. Let $\beta = a_k b_\ell c_{n-k-\ell}$, where $a < b \leq k-1 < k+\ell \leq c$.

Let $\alpha \in (a,a,a)$. Now $\alpha = a_i b_j c_{n-i-j}$, where $a < b < c \leq i-1$. We find

$$\partial_\beta(\alpha) = a_i b_j c_{n-i-j} \cdot a_k b_\ell c_{n-k-\ell} + a_k b_\ell c_{n-k-\ell} \cdot a_i b_j c_{n-i-j} = a_{i+j}c_{n-i-j} + \overline{a} = a_{i+j}c_{n-i-j}.$$

We obtain $\partial_\beta\partial_1(\alpha) = \partial_1(\partial_\beta(\alpha)) = \partial_1(a_{i+j}c_{n-i-j}) = a_{i+j}b_{n-i-j}$. Since $\partial_1(\alpha) = a_ib_{n-i}$, it follows $\beta\partial_1(\alpha) = a_kb_\ell c_{n-k-\ell}\cdot a_ib_{n-i} = \overline{a}$ and $\partial_1(\alpha)\beta = a_ib_{n-i}\cdot a_kb_\ell c_{n-k-\ell} = \overline{a}$. Hence $\partial_1\partial_\beta(\alpha) = \partial_\beta(\partial_1(\alpha)) = \beta\partial_1(\alpha) + \partial_1(\alpha)\beta = \overline{a} < a_{i+j}b_{n-i-j} = \partial_\beta\partial_1(\alpha)$.

Proposition 3. *Let $\alpha \in (a, a, b)$ and $\beta \in (a, a, c)$. Then $\partial_1\partial_\beta(\alpha) \neq \partial_\beta\partial_1(\alpha)$.*

Proof. Now $\alpha = a_ib_jc_{n-i-j}$, where $a < b \leq i-1 < c \leq i+j-1$. We find

$$\partial_\beta(\alpha) = a_ib_jc_{n-i-j}\cdot a_kb_\ell c_{n-k-\ell} + a_kb_\ell c_{n-k-\ell}\cdot a_ib_jc_{n-i-j} = a_{i+j}c_{n-i-j} + a_{k+\ell}b_{n-k-\ell}.$$

Now we obtain $\partial_\beta\partial_1(\alpha) = \partial_1(\partial_\beta(\alpha)) = \partial_1(a_{i+j}c_{n-i-j} + a_{k+\ell}b_{n-k-\ell}) = a_{i+j}b_{n-i-j} + a_{k+\ell}b_{n-k-\ell}$. Since $\partial_1(\alpha) = a_ib_{n-i}$, it follows $\beta\partial_1(\alpha) = a_kb_\ell c_{n-k-\ell}\cdot a_ib_{n-i} = a_{k+\ell}b_{n-k-\ell}$ and $\partial_1(\alpha)\beta = a_ib_{n-i}\cdot a_kb_\ell c_{n-k-\ell} = \overline{b}$. Hence

$$\partial_1\partial_\beta(\alpha) = \partial_\beta(\partial_1(\alpha)) = \beta\partial_1(\alpha) + \partial_1(\alpha)\beta = a_{k+\ell}b_{n-k-\ell} + \overline{b} =$$

$$= \overline{b} > a_{i+j}b_{n-i-j} + a_{k+\ell}b_{n-k-\ell} = \partial_\beta\partial_1(\alpha).$$

Proposition 4. *Let $\alpha \in (a, b, c)$ and $\beta \in (a, a, c)$. Then $\partial_1\partial_\beta(\alpha) \neq \partial_\beta\partial_1(\alpha)$.*

*Proof.*Let $\alpha \in (a, b, c)$. Then $\alpha = a_ib_jc_{n-i-j}$, where $a < i \leq b < i+j \leq c$. Let $\beta = a_kb_\ell c_{n-k-\ell}$, where $a < b \leq k-1 < k+\ell \leq c$.

Now α is a right identity and $\partial_\beta(\alpha) = \alpha\beta + \beta$. But $\alpha\beta = a_ib_jc_{n-i-j}\cdot a_kb_\ell c_{n-k-\ell} = a_{i+j}c_{n-i-j}$. So, $\partial_\beta(\alpha) = a_{i+j}c_{n-i-j} + a_kb_\ell c_{n-k-\ell}$.

We obtain $\partial_\beta\partial_1(\alpha) = \partial_1(\partial_\beta(\alpha)) = \partial_1(a_{i+j}c_{n-i-j} + a_kb_\ell c_{n-k-\ell}) =$

$$= \partial_1(a_{i+j}c_{n-i-j}) + \partial_1(a_kb_\ell c_{n-k-\ell}) = a_{i+j}b_{n-i-j} + a_kb_{n-k}.$$

Since $\partial_1(\alpha) = a_ib_{n-i}$, it follows $\partial_1\partial_\beta(\alpha) = \partial_\beta(\partial_1(\alpha)) =$

$$= \beta\partial_1(\alpha) + \partial_1(\alpha)\beta = a_kb_\ell c_{n-k-\ell}\cdot a_ib_{n-i} + a_ib_{n-i}\cdot a_kb_\ell c_{n-k-\ell} = a_kb_{n-k} + \overline{a} = a_kb_{n-k}.$$

If $i+j < k$, it follows $a_{i+j}b_{n-i-j} > a_kb_{n-k}$. Hence

$$\partial_\beta\partial_1(\alpha) = a_{i+j}b_{n-i-j} > a_kb_{n-k} = \partial_1\partial_\beta(\alpha).$$

Theorem 7. *Let $\alpha \in (a, b, b)$ or $\alpha \in (b, b.b)$, or $\alpha \in (b, b, c)$ and $\beta \in (a, a, c)$. Then $\partial_1\partial_\beta(\alpha) = \partial_\beta\partial_1(\alpha)$.*

Proof. Let $\beta = a_kb_\ell c_{n-k-\ell}$, where $a < b \leq k-1 < k+\ell \leq c$.

Case 1. Let $\alpha \in (a, b, b)$. Now $\alpha = a_ib_jc_{n-i-j}$, $a < i \leq b < c \leq i+j-1$. Then

$$\partial_\beta(\alpha) = a_ib_jc_{n-i-j}\cdot a_kb_\ell c_{n-k-\ell} + a_kb_\ell c_{n-k-\ell}\cdot a_ib_jc_{n-i-j} = a_{i+j}c_{n-i-j} + a_kb_{n-k}.$$

Now we find $\partial_\beta\partial_1(\alpha) = \partial_1(\partial_\beta(\alpha)) = \partial_1(a_{i+j}c_{n-i-j} + a_kb_{n-k}) =$

$$= \partial_1(a_{i+j}c_{n-i-j}) + \partial_(a_kb_{n-k}) = a_{i+j}b_{n-i-j} + a_kb_{n-k}.$$

But $k < c < i+j$. Then $a_k b_{n-k} > a_{i+j} b_{n-i-j}$ and $\partial_\beta \partial_1(\alpha) = a_k b_{n-k}$.
Since $\partial_1(\alpha) = a_i b_{n-i}$, it follows $\partial_1 \partial_\beta(\alpha) = \partial_\beta(\partial_1(\alpha)) = \beta \partial_1(\alpha) + \partial_1(\alpha)\beta =$

$$= a_k b_\ell c_{n-k-\ell} \cdot a_i b_{n-i} + a_i b_{n-i} \cdot a_k b_\ell c_{n-k-\ell} = a_k b_{n-k} + \overline{a} = a_k b_{n-k} = \partial_\beta \partial_1(\alpha).$$

Case 2. Let $\alpha \in (b,b,b)$. Then $\alpha = a_i b_j c_{n-i-j}$, where $i \le a < b < c \le i+j-1$. Now we obtain

$$\partial_\beta(\alpha) = \alpha\beta + \beta\alpha = a_i b_j c_{n-i-j} \cdot a_k b_\ell c_{n-k-\ell} + a_k b_\ell c_{n-k-\ell} \cdot a_i b_j c_{n-i-j} = a_{i+j} c_{n-i-j} + \overline{b}.$$

So, we find $\partial_\beta \partial_1(\alpha) = \partial_1(\partial_\beta(\alpha)) = \partial_1(a_{i+j} c_{n-i-j} + \overline{b}) = a_{i+j} b_{n-i-j} + \overline{b} = \overline{b}$.
Since $\partial_1(\alpha) = a_i b_{n-i} \in (b,b,b)$, it follows

$$\partial_1 \partial_\beta(\alpha) = \partial_\beta(\partial_1(\alpha)) = \beta \partial_1(\alpha) + \partial_1(\alpha)\beta =$$

$$= a_k b_\ell c_{n-k-\ell} \cdot a_i b_{n-i} + a_i b_{n-i} \cdot a_k b_\ell c_{n-k-\ell} = \overline{b} + \overline{a} = \overline{b} = \partial_\beta \partial_1(\alpha).$$

Case 3. Let $\alpha \in (b,b,c)$. Then $\alpha = a_i b_j c_{n-i-j}$, where $i \le a < b \le i+j-1 < c$. We obtain $\partial_\beta(\alpha) = \alpha\beta + \beta\alpha = a_{i+j} c_{n-i-j} + b_{k+\ell} c_{n-k-\ell}$. Thus

$$\partial_\beta \partial_1(\alpha) = \partial_1(\partial_\beta(\alpha)) = \partial_1(a_{i+j} c_{n-i-j} + b_{k+\ell} c_{n-k-\ell}) =$$

$$= \partial_1(a_{i+j} c_{n-i-j}) + \partial_1(b_{k+\ell} c_{n-k-\ell}) = a_{i+j} b_{n-i-j} + \overline{b} = \overline{b}.$$

Since $\partial_1(\alpha) = a_i b_{n-i}$, it follows $\partial_1 \partial_\beta(\alpha) = \partial_\beta(\partial_1(\alpha)) = \partial_1(\alpha)\beta + \beta \partial_1(\alpha) =$

$$= a_i b_{n-i} \cdot a_k b_\ell c_{n-k-\ell} + a_k b_\ell c_{n-k-\ell} \cdot a_i b_{n-i} = \overline{a} + \overline{b} = \overline{b} = \partial_\beta \partial_1(\alpha).$$

3 Conclusions

We prove that $\partial_1 \partial_\beta(\alpha) = \partial_\beta \partial_1(\alpha)$ if

$$\begin{array}{lll}
\beta \in (a,a,a) & \text{and} & \alpha \in \mathcal{TRAP}\{\overline{a}, \overline{b}, b_{b+1} c_{n-b-1}, a_{b+1} c_{n-b-1}\} \\
\beta \in (a,a,b) & \text{and} & \alpha \in (b,b,b),\ \alpha \in (b,b,c) \\
\beta \in (a,b,b) & \text{and} & \alpha \in \mathcal{TRAP}\{\overline{a}, \overline{b}, b_{b+1} c_{n-b-1}, a_{b+1} c_{n-b-1}\} \\
\beta \in (b,b,b) & \text{and} & \alpha \in \mathcal{TRAP}\{\overline{a}, \overline{b}, b_{b+1} c_{n-b-1}, a_{b+1} c_{n-b-1}\}\ . \\
\beta \in (a,a,c) & \text{and} & \alpha \in (a,b,b),\ \alpha \in (b,b.b),\ \alpha \in (b,b,c) \\
\beta \in (a,b,c) & \text{and} & \alpha \in \mathcal{TRAP}\{\overline{a}, \overline{b}, b_{b+1} c_{n-b-1}, a_{b+1} c_{n-b-1}\} \\
\beta \in (b,b,c) & \text{and} & \alpha \in \mathcal{TRAP}\{\overline{a}, \overline{b}, b_{b+1} c_{n-b-1}, a_{b+1} c_{n-b-1}\}
\end{array}$$

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Received 01.09.2017 **Reviewer:** Assoc. prof. PhD Mariana Durcheva

Годишник на Технически Университет - София, том 67, книга 3, 2017
Proceedings of the Technical University of Sofia, Volume 67, Issue 3, 2017

ЙОРДАНОВИ ДИФЕРЕНЦИРАНИЯ, КОМУТИРАЩИ С ПРОЕКЦИЯТА ВЪРХУ СРЕДНИЯ СТРИНГ НА ТРИЪГЪЛНИК

Димитринка Владева

Резюме: *Целта на тази статия е да се намерят Йордановите диференцирания, които комутират с проекцията ∂_2 на $\triangle^{(n)}\{a, b, c\}$ върху средния стринг $\mathcal{STR}^{(n)}\{a, c\}$.*

Ключови думи: *полупръстен от ендоморфизми на крайна верига, диференциална алгебра, Йорданови диференцирания, диференцирания в полупръстени, симплекс.*

JORDAN DERIVATIONS COMMUTING WITH THE PROJECTION ON THE MIDDLE STRING OF A TRIANGLE

Dimitrinka Vladeva

Abstract: *The aim of this paper is to find the Jordan derivations commuting with projection ∂_2 of $\triangle^{(n)}\{a, b, c\}$ on the middle string $\mathcal{STR}^{(n)}\{a, c\}$.*

Keywords: *endomorphism semiring of a finite chain, differential algebra, Jordan derivations, derivations in semirings, simplex.*

1 Introduction

This article and ***Jordan derivation commuting with the projection on the smallest string of a triangle*** and ***Jordan derivation commuting with the projection on the greatest string of a triangle*** published in the same volume, are closely related, that's why we use the introduction and the preliminaries of the first paper and references of the second paper.

2 Jordan derivations commuting with derivation ∂_2

The projection on the middle string of the triangle

$$\partial_2 : \triangle^{(n)}\{a, b, c\} \to \mathcal{STR}^{(n)}\{a, c\}$$

such that for any $\alpha = a_i b_j c_{n-i-j}$,

$$\partial_2(\alpha) = a_i c_{n-i} \in \mathcal{STR}^{(n)}\{a, c\}$$

ISSN 1311-0829

is a derivation – Theorem 3 of [10]. The maximal subsemiring $\mathcal{D}_a$ of the triangle closed under the derivation ∂_2 is consisting of all types except (a, a, b) and (a, a, c) – Theorem 4 of [10].

Theorem 8. *Let $\alpha \in \mathcal{D}_a$ and $\beta \in \mathcal{RI}\left(\triangle^{(n)}\{a, b, c\}\right)$. Then, it follows* $\partial_2\partial_\beta(\alpha) = \partial_\beta\partial_2(\alpha)$.

Proof. Let $\beta = a_k b_\ell c_{n-k-\ell}$, where $a \leq k-1 < b \leq k+\ell-1 < c$ is an element of $\mathcal{RI}\left(\triangle^{(n)}\{a, b, c\}\right)$. Then for any α, it follows that Jordan multiplication

$$\partial_\beta(\alpha) = \alpha\beta + \beta\alpha = \alpha + \beta\alpha$$

is a derivation, see [11].

Case 1. Let $\alpha \in (a, a, a)$. As in case 1 of Theorem 1 we find $\beta\alpha = \overline{a}$ and $\partial_\beta(\alpha) = \alpha$.

Since $\partial_2(\alpha) \in (a, a, a)$, it follows

$$\partial_2\partial_\beta(\alpha) = \partial_\beta(\partial_2(\alpha)) = \partial_2(\alpha) = \partial_2(\partial_\beta(\alpha)) = \partial_\beta\partial_2(\alpha).$$

Case 2. Let $\alpha \in (b, b, b)$. Then $\beta\alpha = \overline{b}$ and then $\partial_\beta(\alpha) = \alpha + \overline{b} = \alpha$. Since $\partial_2(\alpha) \in (c, c, c)$, it follows $\beta\partial_2(\alpha) = \overline{c}$ and

$$\partial_2\partial_\beta(\alpha) = \partial_\beta(\partial_2(\alpha)) = \partial_2(\alpha) + \beta\partial_2(\alpha) = \overline{c}.$$

Hence

$$\partial_\beta\partial_2(\alpha) = \partial_2(\partial_\beta(\alpha)) = \partial_2(\alpha + \overline{b}) = \partial_2(\alpha) + \overline{c} = \overline{c} = \partial_2\partial_\beta(\alpha).$$

Case 3. Let $\alpha \in (b, b, c)$. Then $\alpha = a_i b_j c_{n-i-j}$, where $i \leq a < b \leq i+j-1 < c$. As in case 7 of Theorem 1 we obtain $\beta\alpha = b_{k+\ell} c_{n-k-\ell}$ and $\partial_\beta(\alpha) = \alpha + b_{k+\ell} c_{n-k-\ell}$.

Since $\partial_2(\alpha) = a_i c_{n-i} \in (c, c, c)$, it follows $\beta\partial_2(\alpha) = \overline{c}$. Then we find

$$\partial_2\partial_\beta(\alpha) = \partial_\beta(\partial_2(\alpha)) = \partial_2(\alpha) + \beta\partial_2(\alpha) = \overline{c}.$$

Hence $\partial_\beta\partial_2(\alpha) = \partial_2(\partial_\beta(\alpha)) =$

$$= \partial_2(\alpha + b_{k+\ell} c_{n-k-\ell}) = \partial_2(\alpha) + \partial_2(b_{k+\ell} c_{n-k-\ell}) = \partial_2(\alpha) + \overline{c} = \overline{c} = \partial_2\partial_\beta(\alpha).$$

Case 4. Let $\alpha \in (b, c, c)$. Then $\alpha = a_i b_j c_{n-i-j}$, where $i \leq a < i+j \leq b < c$. We obtain $\beta\alpha = a_k b_\ell c_{n-k-\ell} \cdot a_i b_j c_{n-i-j} = b_k c_{n-k}$ and $\partial_\beta(\alpha) = \alpha + b_k c_{n-k}$. As in the previous case $\partial_2(\alpha) = a_i c_{n-i} \in (c, c, c)$ and $\beta\partial_2(\alpha) = \overline{c}$. So, we find

$$\partial_2\partial_\beta(\alpha) = \partial_\beta(\partial_2(\alpha)) = \partial_2(\alpha) + \beta\partial_2(\alpha) = \overline{c}.$$

Hence

$$\partial_\beta\partial_2(\alpha) = \partial_2(\partial_\beta(\alpha)) = \partial_2(\alpha + b_k c_{n-k}) = \partial_2(\alpha) + \partial_2(b_{k+} c_{n-k}) = \overline{c} = \partial_2\partial_\beta(\alpha).$$

Case 5. Let $\alpha \in (c, c, c)$. Then $\beta\alpha = \overline{c}$ and $\partial_\beta(\alpha) = \overline{c}$. Since $\partial_2(\alpha) \in (c, c, c)$, it follows $\partial_2\partial_\beta(\alpha) = \partial_\beta(\partial_2(\alpha)) = \overline{c}$. Hence

$$\partial_\beta\partial_2(\alpha) = \partial_2(\partial_\beta(\alpha)) = \partial_2(\overline{c}) = \overline{c} = \partial_2\partial_\beta(\alpha).$$

Case 6. Let $\alpha \in (a, c, c)$. Then $\alpha = a_i b_j c_{n-i-j}$, where $a \le i - 1 < i + j \le b < c$. We obtain $\beta\alpha = a_k b_\ell c_{n-k-\ell} \cdot a_i b_j c_{n-i-j} = a_k c_{n-k}$ and then $\partial_\beta(\alpha) = \alpha + a_k c_{n-k}$. Since $\partial_2(\alpha) \in (a, c, c)$, it follows $\partial_2\partial_\beta(\alpha) = \partial_\beta(\partial_2(\alpha)) = \partial_2(\alpha) + a_k c_{n-k}$. Hence

$$\partial_\beta\partial_2(\alpha) = \partial_2(\partial_\beta(\alpha)) = \partial_2(\alpha + a_k c_{n-k}) =$$

$$= \partial_2(\alpha) + \partial_2(a_{k+} c_{n-k}) = \partial_2(\alpha) + a_k c_{n-k} = \partial_2\partial_\beta(\alpha).$$

Case 7. Let $\alpha \in (a, b, c)$. Then α is a right identity and $\partial_\beta(\alpha) = \alpha + \beta$. Thus

$$\partial_\beta\partial_2(\alpha) = \partial_2(\partial_\beta(\alpha)) = \partial_2(\alpha + \beta) = \partial_2(\alpha) + \partial_2(\beta) = \partial_2(\alpha) + a_k c_{n-k}.$$

Since $\partial_2(\alpha) = a_i c_{n-i}$, it follows $\beta\partial_2(\alpha) = a_k b_\ell c_{n-k-\ell} \cdot a_i c_{n-i} = a_k c_{n-k}$. Hence,

$$\partial_2\partial_\beta(\alpha) = \partial_\beta(\partial_1(\alpha)) = \partial_2(\alpha) + \beta\partial_2(\alpha) = \partial_2(\alpha) + a_k c_{n-k} = \partial_\beta\partial_2(\alpha).$$

Case 8. Let $\alpha \in (a, b, b)$. Then $\alpha = a_i b_j c_{n-i-j}$, where $a \le i-1 < b < c \le i+j-1$. As in case 3 of Theorem 1 we obtain $\beta\alpha = a_k b_{n-k}$ and then $\partial_\beta(\alpha) = \alpha + a_k b_{n-k}$.

Since $\partial_2(\alpha) = a_i c_{n-i}$, it follows $\beta\partial_2(\alpha) = a_k b_\ell c_{n-k-\ell} \cdot a_i c_{n-i} = a_k c_{n-k}$. Thus, we have

$$\partial_\beta\partial_2(\alpha) = \partial_2(\partial_\beta(\alpha))\partial_2(\alpha + a_k b_{n-k}) = \partial_2(\alpha) + \partial_2(a_k b_{n-k}) = \partial_2(\alpha) + a_k c_{n-k}.$$

Hence

$$\partial_2\partial_\beta(\alpha) = \partial_\beta(\partial_2(\alpha)) = \partial_2(\alpha) + \beta\partial_2(\alpha) = \partial_2(\alpha) + a_k c_{n-k} = \partial_\beta\partial_2(\alpha).$$

Theorem 9. *Let $\alpha \in \mathcal{D}_a$ and $\beta \in (a, a, a)$. Then $\partial_2\partial_\beta(\alpha) = \partial_\beta\partial_2(\alpha)$.*

Proof. Case 1. Let $\alpha(a) = a$. Then $\beta\alpha = \overline{a}$ and $\partial_\beta(\alpha) = \alpha\beta + \beta\alpha = \overline{a} + \overline{a} = \overline{a}$. Thus

$$\partial_\beta\partial_2(\alpha) = \partial_2(\partial_\beta(\alpha)) = \partial_2(\overline{a}) = \overline{a}.$$

Since $\partial_2(\alpha)(a) = a$, it follows

$$\partial_2\partial_\beta(\alpha) = \partial_\beta(\partial_2(\alpha)) = \partial_\beta(\overline{a}) = \overline{a} = \partial_\beta\partial_2(\alpha).$$

Case 2. Let $\alpha(a) = b$. Then $\beta\alpha = \overline{b}$ and $\partial_\beta(\alpha) = \alpha\beta + \beta\alpha = \overline{a} + \overline{b} = \overline{b}$. Now

$$\partial_\beta\partial_2(\alpha) = \partial_2(\partial_\beta(\alpha)) = \partial_2(\overline{b}) = \overline{c}.$$

Since $\partial_2(\alpha) \in (c, c, c)$, it follows $\beta\partial_2(\alpha) = \overline{c}$ and then

$$\partial_2\partial_\beta(\alpha) = \partial_\beta(\partial_2(\alpha)) = \beta\partial_2(\alpha) + \partial_2(\alpha)\beta = \overline{c} + \overline{a} = \overline{c} = \partial_\beta\partial_2(\alpha).$$

Not all Jordan derivations commutes with the derivation ∂_2 as we now show.

Proposition 5. *Let $\alpha \in (a, a, a)$ and $\beta \in (b, b, b)$. Then $\partial_2\partial_\beta(\alpha) \neq \partial_\beta\partial_2(\alpha)$.*

Proof. We obtain $\partial_\beta(\alpha) = \alpha\beta + \beta\alpha = \overline{b} + \overline{a} = \overline{b}$. Then

$$\partial_\beta\partial_2(\alpha) = \partial_2(\partial_\beta(\alpha)) = \partial_2(\overline{b}) = \overline{c}.$$

Since $\partial_2(\alpha) \in (a, a, a)$, it follows $\beta\partial_2(\alpha) = \overline{a}$. Now, we observe

$$\partial_2\partial_\beta(\alpha) = \partial_\beta(\partial_2(\alpha)) = \beta\partial_2(\alpha) + \partial_2(\alpha)\beta = \overline{a} + \overline{b} = \overline{b} < \overline{c} = \partial_\beta\partial_2(\alpha).$$

Theorem 10. *Let $\alpha \in \mathcal{D}_a$, $\alpha \notin (a, a, a)$ and $\beta \in (b, b, b)$. Then $\partial_2\partial_\beta(\alpha) = \partial_\beta\partial_2(\alpha)$.*

Proof. Case 1. Let $\alpha(b) = b$. Then $\partial_\beta(\alpha) = \alpha\beta + \beta\alpha = \overline{b} + \overline{b} = \overline{b}$. Now we have

$$\partial_\beta\partial_2(\alpha) = \partial_2(\partial_\beta(\alpha)) = \partial_2(\overline{b}) = \overline{c}.$$

Since $\partial_2(\alpha)(b) = c$, it follows $\beta\partial_2(\alpha) = \overline{c}$. So

$$\partial_2\partial_\beta(\alpha) = \partial_\beta(\partial_2(\alpha)) = \beta\partial_2(\alpha) + \partial_2(\alpha)\beta = \overline{c} + \overline{b} = \overline{c} = \partial_\beta\partial_2(\alpha).$$

Case 2. Let $\alpha(b) = c$. Then $\partial_\beta(\alpha) = \alpha\beta + \beta\alpha = \overline{b} + \overline{c} = \overline{c}$. We obtain

$$\partial_\beta\partial_2(\alpha) = \partial_2(\partial_\beta(\alpha)) = \partial_2(\overline{c}) = \overline{c}.$$

Since $\partial_2(\alpha)(b) = c$, it follows $\beta\partial_2(\alpha) = \overline{c}$. So

$$\partial_2\partial_\beta(\alpha) = \partial_\beta(\partial_2(\alpha)) = \beta\partial_2(\alpha) + \partial_2(\alpha)\beta = \overline{c} + \overline{b} = \overline{c} = \partial_\beta\partial_2(\alpha).$$

Theorem 11. *Let $\alpha \in \mathcal{D}_a$ and $\beta \in (c, c, c)$. Then $\partial_2\partial_\beta(\alpha) = \partial_\beta\partial_2(\alpha)$.*

Proof. We obtain $\partial_\beta(\alpha) = \alpha\beta + \beta\alpha = \overline{c} + \beta\alpha = \overline{c}$. Then we find

$$\partial_\beta\partial_2(\alpha) = \partial_2(\partial_\beta(\alpha)) = \partial_2(\overline{c}) = \overline{c}.$$

Hence

$$\partial_2\partial_\beta(\alpha) = \partial_\beta(\partial_2(\alpha)) = \beta\partial_2(\alpha) + \partial_2(\alpha)\beta = \beta\partial_2(\alpha) + \overline{c} = \overline{c} = \partial_\beta\partial_2(\alpha).$$

If we choose β to be an element of another type of the triangle, we obtain more restrictions for endomorphism α, as we now show.

Theorem 12. *Let $\alpha \in \mathcal{D}_a$, $\alpha \notin (a,a,a)$, $\alpha \notin (a,b,b)$ and $\beta \in (b,c,c)$. Then $\partial_2\partial_\beta(\alpha) = \partial_\beta\partial_2(\alpha)$.*

Proof. Case 1. Let $\alpha(a) = b$. Then $\alpha\beta = \overline{c}$ and then $\partial_\beta(\alpha) = \overline{c} + \beta\alpha = \overline{c}$. We obtain

$$\partial_\beta\partial_2(\alpha) = \partial_2(\partial_\beta(\alpha)) = \partial_2(\overline{c}) = \overline{c}.$$

Since $\partial_2(\alpha) \in (c,c,c)$, it follows $\beta\partial_2(\alpha) = \overline{c}$. So

$$\partial_2\partial_\beta(\alpha) = \partial_\beta(\partial_2(\alpha)) = \beta\partial_2(\alpha) + \partial_2(\alpha)\beta = \overline{c} + \partial_2(\alpha)\beta = \overline{c} = \partial_\beta\partial_2(\alpha).$$

Case 2. Let $\alpha(b) = c$. Then $\beta\alpha = \overline{c}$ and then $\partial_\beta(\alpha) = \alpha\beta + \overline{c} = \overline{c}$. We obtain

$$\partial_\beta\partial_2(\alpha) = \partial_2(\partial_\beta(\alpha)) = \partial_2(\overline{c}) = \overline{c}.$$

Since $\partial_2(\alpha)(b) = c$, it follows $\beta\partial_2(\alpha) = \overline{c}$. So

$$\partial_2\partial_\beta(\alpha) = \partial_\beta(\partial_2(\alpha)) = \beta\partial_2(\alpha) + \partial_2(\alpha)\beta = \overline{c} + \partial_2(\alpha)\beta = \overline{c} = \partial_\beta\partial_2(\alpha).$$

Case 3. Let $\alpha \in (a, b.c)$. Then $\partial_\beta(\alpha) = \alpha\beta + \beta\alpha = \alpha\beta + \beta$.
Now we obtain

$$\partial_\beta\partial_2(\alpha) = \partial_2(\partial_\beta(\alpha)) = \partial_2(\alpha\beta + \beta) = \partial_2(\alpha\beta) + \partial_2(\beta) = \partial_2(\alpha) + \overline{c} = \overline{c}.$$

Since $\partial_2(\alpha) \in (a,c,c)$, as in the previous case we find $\beta\partial_2(\alpha) = \overline{c}$. Hence

$$\partial_2\partial_\beta(\alpha) = \partial_\beta(\partial_2(\alpha)) = \beta\partial_2(\alpha) + \partial_2(\alpha)\beta = \overline{c} + \partial_2(\alpha)\beta = \overline{c} = \partial_\beta\partial_2(\alpha).$$

Proposition 6. *Let $\alpha \in (a,a,a)$ and $\beta \in (a,b,b)$. Then $\partial_2\partial_\beta(\alpha) \neq \partial_\beta\partial_2(\alpha)$.*

Proof. Let $\alpha \in (a,a,a)$. Then $\alpha = a_i b_j c_{n-i-j}$, where $a < b < c \leq i-1$. Let $\beta = a_k b_\ell c_{n-k-\ell}$, where $a \leq k-1 < b < c \leq k+\ell-1$. We find

$$\partial_\beta(\alpha) = \alpha\beta + \beta\alpha = a_i b_j c_{n-i-j} \cdot a_k b_\ell c_{n-k-\ell} + a_k b_\ell c_{n-k-\ell} \cdot a_i b_j c_{n-i-j} =$$

$$= a_i b_{n-i} + \overline{a} = a_i b_{n-i}.$$

We obtain

$$\partial_\beta\partial_2(\alpha) = \partial_2(\partial_\beta(\alpha)) = \partial_2(a_i b_{n-i}) = a_i c_{n-i}.$$

Since $\partial_2(\alpha) = a_i c_{n-i}$, it follows $\beta\partial_2(\alpha) = a_k b_\ell c_{n-k-\ell} \cdot a_i c_{n-i} = \overline{a}$ and $\partial_2(\alpha)\beta = a_i c_{n-i} \cdot a_k b_\ell c_{n-k-\ell} = a_i b_{n-i}$. Hence

$$\partial_2\partial_\beta(\alpha) = \partial_\beta(\partial_2(\alpha)) = \beta\partial_2(\alpha) + \partial_2(\alpha)\beta =$$

$$= \overline{a} + a_i b_{n-i} = a_i b_{n-i} < a_i c_{n-i} = \partial_\beta\partial_2(\alpha).$$

Proposition 7. *Let $\alpha \in (a, b, b)$ and $\beta \in (a, b, b)$. Then $\partial_2\partial_\beta(\alpha) \neq \partial_\beta\partial_2(\alpha)$.*

Proof. Let $\alpha \in (a, b, b)$. Then $\alpha = a_i b_j c_{n-i-j}$, where $a \leq i-1 < b < c \leq i+j-1$. Let $\beta = a_k b_\ell c_{n-k-\ell}$, where $a \leq k-1 < b < c \leq k+\ell-1$. As in the previous proposition we find $\partial_\beta(\alpha) = a_i b_{n-i} + a_k b_{n-k}$.

Now we obtain

$$\partial_\beta\partial_2(\alpha) = \partial_2(\partial_\beta(\alpha)) = \partial_2(a_i b_{n-i} + a_k b_{n-k}) = a_i c_{n-i} + a_k c_{n-k}.$$

Since $\partial_2(\alpha) = a_i c_{n-i}$, it follows $\beta\partial_2(\alpha) = a_k b_\ell c_{n-k-\ell} \cdot a_i c_{n-i} = a_k c_{n-k}$ and $\partial_2(\alpha)\beta = a_i c_{n-i} \cdot a_k b_\ell c_{n-k-\ell} = a_i b_{n-i}$. Hence

$$\partial_2\partial_\beta(\alpha) = \partial_\beta(\partial_2(\alpha)) = \beta\partial_2(\alpha) + \partial_2(\alpha)\beta = a_k c_{n-k} + a_i b_{n-i}.$$

If $i \leq k$ we obtain $a_i c_{n-i} \geq a_k c_{n-k}$ and then $\partial_\beta\partial_2(\alpha) = a_i c_{n-i}$. But now

$$\partial_2\partial_\beta(\alpha) = a_k c_{n-k} + a_i b_{n-i} = a_i bk - i c_{n-i} < a_i c_{n-i} = \partial_\beta\partial_2(\alpha).$$

By similar arguments we can prove that if $\alpha \in (a, b, c)$ or $\alpha \in (a, c, c)$ and $\beta \in (a, b, b)$ then it follows $\partial_2\partial_\beta(\alpha) \neq \partial_\beta\partial_2(\alpha)$.

Theorem 13. *Let $\alpha \in \mathcal{D}_a$, $\alpha(a) \geq b$ and $\beta \in (a, b, b)$. Then, it follows $\partial_2\partial_\beta(\alpha) = \partial_\beta\partial_2(\alpha)$.*

Proof. We obtain

$$\begin{array}{c} \alpha\beta(a) = \beta(\alpha(a)) = \alpha(a) = \alpha(\beta(a)) = \beta\alpha(a) \\ \alpha\beta(b) = \beta(\alpha(b)) = b \leq \alpha(b) = \alpha(\beta(b)) = \beta\alpha(b) \\ \alpha\beta(c) = \beta(\alpha(c)) = b \leq \alpha(b)` = \alpha(\beta(c)) = \beta\alpha(c) \end{array}.$$

Thus $\partial_\beta(\alpha) = \beta\alpha$. So, $\partial_\beta\partial_2(\alpha) = \partial_2(\partial_\beta(\alpha)) = \partial_2(\alpha\beta) = \partial_2(\alpha)\beta + \beta\partial_2(\alpha)$. Since $\partial_2(\alpha) \in (c, c, c)$, it follows $\beta\partial_2(\alpha) = \overline{c}$ and then $\partial_\beta\partial_2(\alpha) = \overline{c}$. Hence

$$\partial_2\partial_\beta(\alpha) = \partial_\beta(\partial_2(\alpha)) = \beta\partial_2(\alpha) + \partial_2(\alpha)\beta = \overline{c} + \partial_2(\alpha)\beta = \overline{c} = \partial_\beta\partial_2(\alpha).$$

Proposition 8. *Let $\alpha \in (a,a,a)$ and $\beta \in (b,b,c)$. Then $\partial_2\partial_\beta(\alpha) \neq \partial_\beta\partial_2(\alpha)$.*

Proof. Let $\alpha \in (a,a,a)$. Then $\alpha = a_i b_j c_{n-i-j}$, where $a < b < c \leq i-1$. Let $\beta = a_k b_\ell c_{n-k-\ell}$, where $k \leq a < b \leq k+\ell-1 < c$. We find

$$\partial_\beta(\alpha) = \alpha\beta + \beta\alpha = a_i b_j c_{n-i-j} \cdot a_k b_\ell c_{n-k-\ell} + a_k b_\ell c_{n-k-\ell} \cdot a_i b_j c_{n-i-j} =$$

$$= b_{i+j} c_{n-i-j} + \overline{a} = b_{i+j} c_{n-i-j}.$$

Thus

$$\partial_\beta\partial_2(\alpha) = \partial_2(\partial_\beta(\alpha)) = \partial_2(b_{i+j} c_{n-i-j}) = \overline{c}.$$

Since $\partial_2(\alpha) = a_i c_{n-i}$, it follows $\beta\partial_2(\alpha) = a_k b_\ell c_{n-k-\ell} \cdot a_i c_{n-i} = \overline{a}$ and $\partial_2(\alpha)\beta = a_i c_{n-i} \cdot a_k b_\ell c_{n-k-\ell} = b_i c_{n-i}$.

Hence

$$\partial_2\partial_\beta(\alpha) = \partial_\beta(\partial_2(\alpha)) = \beta\partial_2(\alpha) + \partial_2(\alpha)\beta = \overline{a} + b_i c_{n-i} = b_i c_{n-i} < \overline{c} = \partial_\beta\partial_2(\alpha).$$

By similar arguments we can prove that if $\alpha \in (a,b,b)$ or $\alpha \in (a,b,c)$, or $\alpha \in (a,c,c)$ and $\beta \in (b,b,c)$ then it follows $\partial_2\partial_\beta(\alpha) \neq \partial_\beta\partial_2(\alpha)$.

Theorem 14. *Let $\alpha \in \mathcal{D}_a$, $\alpha(a) \geq b$ and $\beta \in (b,b,c)$. Then, it follows $\partial_2\partial_\beta(\alpha) = \partial_\beta\partial_2(\alpha)$.*

Proof. Let $\beta = a_k b_\ell c_{n-k-\ell}$, where $k \leq a < b \leq k+\ell-1 < c$. We obtain

$$\begin{array}{c} \alpha\beta(a) = \beta(\alpha(a)) = \alpha(a) \leq \alpha(b) = \alpha(\beta(a)) = \beta\alpha(a) \\ \alpha\beta(b) = \beta(\alpha(b)) = \alpha(b) = \alpha(\beta(b)) = \beta\alpha(b) \\ \alpha\beta(c) = \beta(\alpha(c)) = \alpha(c) = \alpha(\beta(c)) = \beta\alpha(c) \end{array}.$$

Now, as in Theorem 13 we obtain $\partial_\beta(\alpha) = \beta\alpha$. So, $\partial_\beta\partial_2(\alpha) = \partial_2(\partial_\beta(\alpha)) = \partial_2(\alpha\beta) = \partial_2(\alpha)\beta + \beta\partial_2(\alpha)$.

Since $\partial_2(\alpha) \in (c,c,c)$, it follows $\beta\partial_2(\alpha) = \overline{c}$ and then we obtain $\partial_\beta\partial_2(\alpha) = \overline{c}$.

Hence

$$\partial_2\partial_\beta(\alpha) = \partial_\beta(\partial_2(\alpha)) = \beta\partial_2(\alpha) + \partial_2(\alpha)\beta = \overline{c} + \partial_2(\alpha)\beta = \overline{c} = \partial_\beta\partial_2(\alpha).$$

Theorem 15. *Let $\alpha \in \mathcal{D}_a$ and $\beta \in (a,c,c)$. Then $\partial_2\partial_\beta(\alpha) = \partial_\beta\partial_2(\alpha)$.*

Proof. Let $\beta = a_k b_\ell c_{n-k-\ell}$, where $a \leq k-1 < k+\ell \leq b < c$.

Case 1. Let $\alpha \in (a,a,a)$. Then $\alpha = a_i b_j c_{n-i-j}$, where $a < b < c \leq i-1$.

We obtain

$$\partial_\beta(\alpha) = \alpha\beta + \beta\alpha = a_i b_j c_{n-i-j} \cdot a_k b_\ell c_{n-k-\ell} + a_k b_\ell c_{n-k-\ell} \cdot a_i b_j c_{n-i-j} =$$

$$= a_i c_{n-i} + \overline{a} = a_i c_{n-i}.$$

Thus

$$\partial_\beta \partial_2(\alpha) = \partial_2(\partial_\beta(\alpha)) = \partial_2(a_i c_{n-i}) = a_i c_{n-i}.$$

Since $\partial_2(\alpha) = a_i c_{n-i}$, it follows $\beta\partial_2(\alpha) = a_k b_\ell c_{n-k-\ell} \cdot a_i c_{n-i} = \overline{a}$ and $\partial_2(\alpha)\beta = a_i c_{n-i} \cdot a_k b_\ell c_{n-k-\ell} = a_i c_{n-i}$. Hence

$$\partial_2 \partial_\beta(\alpha) = \partial_\beta(\partial_2(\alpha)) = \beta\partial_2(\alpha) + \partial_2(\alpha)\beta = \overline{a} + a_i c_{n-i} = a_i c_{n-i} < \overline{c} = \partial_\beta \partial_2(\alpha).$$

Case 2. Let $\alpha \in (a, a, b)$. Then $\alpha = a_i b_j c_{n-i-j}$, where $a < b \leq i-1 < c \leq i+j-1$. We obtain

$$\partial_\beta(\alpha) = \alpha\beta + \beta\alpha = a_i b_j c_{n-i-j} \cdot a_k b_\ell c_{n-k-\ell} + a_k b_\ell c_{n-k-\ell} \cdot a_i b_j c_{n-i-j} =$$

$$= a_i c_{n-i} + a_{k+\ell} b_{n-k-\ell}.$$

Thus $\partial_\beta \partial_2(\alpha) = \partial_2(\partial_\beta(\alpha)) = \partial_2(a_i c_{n-i} + a_{k+\ell} b_{n-k-\ell}) =$

$$= \partial_2(a_i c_{n-i}) + \partial_2(a_{k+\ell} b_{n-k-\ell}) = a_i c_{n-i} + a_{k+\ell} c_{n-k-\ell}.$$

But $k + \ell \leq b < i$. So, $a_i c_{n-i} < a_{k+\ell} c_{n-k-\ell}$ and $\partial_\beta \partial_2(\alpha) = a_{k+\ell} c_{n-k-\ell}$.

Since $\partial_2(\alpha) = a_i c_{n-i}$, it follows $\beta\partial_2(\alpha) = a_k b_\ell c_{n-k-\ell} \cdot a_i c_{n-i} = a_{k+\ell} c_{n-k-\ell}$ and $\partial_2(\alpha)\beta = a_i c_{n-i} \cdot a_k b_\ell c_{n-k-\ell} = a_i c_{n-i}$.

Hence

$$\partial_2 \partial_\beta(\alpha) = \partial_\beta(\partial_2(\alpha)) = \beta\partial_2(\alpha) + \partial_2(\alpha)\beta =$$

$$= a_{k+\ell} c_{n-k-\ell} + a_i c_{n-i} = a_{k+\ell} c_{n-k-\ell} = \partial_\beta \partial_2(\alpha).$$

Case 3. Let $\alpha \in (a, b, b)$. Then $\alpha = a_i b_j c_{n-i-j}$, where $a \leq i-1 < b < c \leq i+j-1$. We obtain

$$\partial_\beta(\alpha) = \alpha\beta + \beta\alpha = a_i b_j c_{n-i-j} \cdot a_k b_\ell c_{n-k-\ell} + a_k b_\ell c_{n-k-\ell} \cdot a_i b_j c_{n-i-j} = a_i c_{n-i} + a_k b_{n-k}.$$

Thus $\partial_\beta \partial_2(\alpha) = \partial_2(\partial_\beta(\alpha)) = \partial_2(a_i c_{n-i} + a_k b_{n-k}) =$

$$= \partial_2(a_i c_{n-i}) + \partial_2(a_k b_{n-k}) = a_i c_{n-i} + a_k c_{n-k}.$$

Since $\partial_2(\alpha) = a_i c_{n-i}$, it follows $\beta\partial_2(\alpha) = a_k b_\ell c_{n-k-\ell} \cdot a_i c_{n-i} = a_k c_{n-k}$ and $\partial_2(\alpha)\beta = a_i c_{n-i} \cdot a_k b_\ell c_{n-k-\ell} = a_i c_{n-i}$. Hence

$$\partial_2 \partial_\beta(\alpha) = \partial_\beta(\partial_2(\alpha)) = \beta\partial_2(\alpha) + \partial_2(\alpha)\beta = a_k c_{n-k} + a_i c_{n-i} = \partial_\beta \partial_2(\alpha).$$

Case 4. Let $\alpha \in (b, b, b)$. Then $\alpha = a_i b_j c_{n-i-j}$, where $i \leq a < b < c \leq i + j - 1$.

We obtain

$$\partial_\beta(\alpha) = \alpha\beta + \beta\alpha = a_i b_j c_{n-i-j} \cdot a_k b_\ell c_{n-k-\ell} + a_k b_\ell c_{n-k-\ell} \cdot a_i b_j c_{n-i-j} = a_i c_{n-i} + \overline{b}.$$

Thus $\partial_\beta\partial_2(\alpha) = \partial_2(\partial_\beta(\alpha)) = \partial_2(a_i c_{n-i} + \overline{b}) =$

$$= \partial_2(a_i c_{n-i}) + \partial_2(\overline{b}) = a_i c_{n-i} + \overline{c} = \overline{c}.$$

Since $\partial_2(\alpha) = a_i c_{n-i}$, it follows $\beta\partial_2(\alpha) = a_k b_\ell c_{n-k-\ell} \cdot a_i c_{n-i} = \overline{c}$ and $\partial_2(\alpha)\beta = a_i c_{n-i} \cdot a_k b_\ell c_{n-k-\ell} = a_i c_{n-i}$.

Hence

$$\partial_2\partial_\beta(\alpha) = \partial_\beta(\partial_2(\alpha)) = \beta\partial_2(\alpha) + \partial_2(\alpha)\beta = \overline{c} + a_i c_{n-i} = \overline{c} = \partial_\beta\partial_2(\alpha).$$

Case 5. Let $\alpha \in (a, b, c)$. Then $\alpha = a_i b_j c_{n-i-j}$, where $a \leq i-1 < b \leq i+j-1 < c$.

Since α is a right identity, it follows

$$\partial_\beta(\alpha) = \alpha\beta + \beta = a_i b_j c_{n-i-j} \cdot a_k b_\ell c_{n-k-\ell} + a_k b_\ell c_{n-k-\ell} = a_i c_{n-i} + a_k b_\ell c_{n-k-\ell}.$$

Thus $\partial_\beta\partial_2(\alpha) = \partial_2(\partial_\beta(\alpha)) = \partial_2(a_i c_{n-i} + a_k b_\ell c_{n-k-\ell}) =$

$$= \partial_2(a_i c_{n-i}) + \partial_2(a_k b_\ell c_{n-k-\ell}) = a_i c_{n-i} + a_k c_{n-k} = a_i c_{n-i} + a_k c_{n-k}.$$

Since $\partial_2(\alpha) = a_i c_{n-i}$, it follows $\beta\partial_2(\alpha) = a_k b_\ell c_{n-k-\ell} \cdot a_i c_{n-i} = a_k c_{n-k}$ and $\partial_2(\alpha)\beta = a_i c_{n-i} \cdot a_k b_\ell c_{n-k-\ell} = a_i c_{n-i}$. Hence

$$\partial_2\partial_\beta(\alpha) = \partial_\beta(\partial_2(\alpha)) = \beta\partial_2(\alpha) + \partial_2(\alpha)\beta = a_k c_{n-k} + a_i c_{n-i} = \partial_\beta\partial_2(\alpha).$$

Case 6. Let $\alpha \in (b, b, c)$. Then $\alpha = a_i b_j c_{n-i-j}$, where $i \leq a < b \leq i + j - 1 < c$. We obtain

$$\partial_\beta(\alpha) = \alpha\beta + \beta = a_i b_j c_{n-i-j} \cdot a_k b_\ell c_{n-k-\ell} + a_k b_\ell c_{n-k-\ell} = a_i c_{n-i} + b_{k+\ell} c_{n-k-\ell}.$$

Thus $\partial_\beta\partial_2(\alpha) = \partial_2(\partial_\beta(\alpha)) = \partial_2(a_i c_{n-i} + b_{k+\ell} c_{n-k-\ell}) =$

$$= \partial_2(a_i c_{n-i}) + \partial_2(b_{k+\ell} c_{n-k-\ell}) = a_i c_{n-i} + \overline{c} = \overline{c}.$$

Since $\partial_2(\alpha) = a_i c_{n-i}$, it follows $\beta\partial_2(\alpha) = a_k b_\ell c_{n-k-\ell} \cdot a_i c_{n-i} = \overline{c}$ and $\partial_2(\alpha)\beta = a_i c_{n-i} \cdot a_k b_\ell c_{n-k-\ell} = a_i c_{n-i}$.

Hence

$$\partial_2\partial_\beta(\alpha) = \partial_\beta(\partial_2(\alpha)) = \beta\partial_2(\alpha) + \partial_2(\alpha)\beta = \overline{c} + a_i c_{n-i} = \overline{c} = \partial_\beta\partial_2(\alpha).$$

Case 7. Let $\alpha \in (a, a, c)$. Then $\alpha = a_i b_j c_{n-i-j}$, where $a < b \leq i-1 < i+j \leq c$. We obtain

$$\partial_\beta(\alpha) = \alpha\beta + \beta = a_i b_j c_{n-i-j} \cdot a_k b_\ell c_{n-k-\ell} + a_k b_\ell c_{n-k-\ell} = a_i c_{n-i} + a_{k+\ell} c_{n-k-\ell}.$$

But $k + \ell \leq b < i$ implies $a_i c_{n-i} < a_{k+\ell} c_{n-k-\ell}$ and $\partial_\beta(\alpha) = a_{k+\ell} c_{n-k-\ell}$.

Thus $\partial_\beta \partial_2(\alpha) = \partial_2(\partial_\beta(\alpha)) = \partial_2(a_{k+\ell} c_{n-k-\ell}) = a_{k+\ell} c_{n-k-\ell}$.

Since $\partial_2(\alpha) = a_i c_{n-i}$, it follows $\beta\partial_2(\alpha) = a_k b_\ell c_{n-k-\ell} \cdot a_i c_{n-i} = a_{k+\ell} c_{n-k-\ell}$ and $\partial_2(\alpha)\beta = a_i c_{n-i} \cdot a_k b_\ell c_{n-k-\ell} = a_i c_{n-i}$.

Hence

$$\partial_2 \partial_\beta(\alpha) = \partial_\beta(\partial_2(\alpha)) = \beta\partial_2(\alpha) + \partial_2(\alpha)\beta =$$
$$= a_{k+\ell} c_{n-k-\ell} + a_i c_{n-i} = a_{k+\ell} c_{n-k-\ell} = \partial_\beta \partial_2(\alpha).$$

Case 8. Let $\alpha \in (c, c, c)$. Then $\beta\alpha = \overline{c}$ and $\partial_\beta(\alpha) = \overline{c}$. Thus $\partial_\beta \partial_2(\alpha) = \partial_2(\partial_\beta(\alpha)) = \partial_2(\overline{c}) = \overline{c}$. Since $\partial_2(\alpha) \in (c, c, c)$, it follows $\beta\partial_2(\alpha) = \overline{c}$.

Hence

$$\partial_2 \partial_\beta(\alpha) = \partial_\beta(\partial_2(\alpha)) = \beta\partial_2(\alpha) + \partial_2(\alpha)\beta =$$
$$= \overline{c} + \partial_2(\alpha)\beta = \partial_2(\alpha)\beta = \partial_\beta \partial_2(\alpha).$$

3 Conclusions

Here we prove that $\partial_2 \partial_\beta(\alpha) = \partial_\beta \partial_2(\alpha)$ if

$$\begin{array}{lll}
\beta \in (a, a, a) & \text{and} & \alpha \in \mathcal{D}_a \\
\beta \in (b, b, b) & \text{and} & \alpha \in \mathcal{D}_a \backslash (a, a, a) \\
\beta \in (b, b, c) & \text{and} & \alpha \in (b, b, b),\ \alpha \in (b, b, c),\ \alpha \in (b, c, c),\ \alpha \in (c, c, c) \\
\beta \in (b, c, c) & \text{and} & \alpha \in \mathcal{D}_a \backslash \{(a, a, a), (a, b, b)\} \\
\beta \in (c, c, c) & \text{and} & \alpha \in \mathcal{D}_a \\
\beta \in (a, b, b) & \text{and} & \alpha \in (b, b, b),\ \alpha \in (b, b, c),\ \alpha \in (b, c, c),\ \alpha \in (c, c, c) \\
\beta \in (a, b, c) & \text{and} & \alpha \in \mathcal{D}_a \\
\beta \in (a, c, c) & \text{and} & \alpha \in \mathcal{D}_a
\end{array}.$$

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Received 01.09.2017 **Reviewer:** Assoc. prof. PhD Mariana Durcheva

Годишник на Технически Университет - София, том 67, книга 3, 2017
Proceedings of the Technical University of Sofia, Volume 67, Issue 3, 2017

ЙОРДАНОВИ ДИФЕРЕНЦИРАНИЯ, КОМУТИРАЩИ С ПРОЕКЦИЯТА ВЪРХУ НАЙ-ГОЛЕМИЯ СТРИНГ НА ТРИЪГЪЛНИК

Димитринка Владева

Резюме: *Целта на тази статия е да се намерят Йордановите диференцирания, които комутират с проекцията ∂_3 на $\triangle^{(n)}\{a,b,c\}$ върху $\mathcal{STR}^{(n)}\{b,c\}$.*

Ключови думи: *полупръстен от ендоморфизми на крайна верига, диференциална алгебра, Йорданови диференцирания, диференцирания в полупръстени.*

JORDAN DERIVATIONS COMMUTING WITH THE PROJECTION ON THE GREATEST STRING OF A TRIANGLE

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Abstract: *The aim of this paper is to find the Jordan derivations commuting with projection ∂_3 of $\triangle^{(n)}\{a,b,c\}$ on the greatest string $\mathcal{STR}^{(n)}\{b,c\}$.*

Keywords: *endomorphism semiring of a finite chain, differential algebra, Jordan derivations, derivations in semirings.*

1 Introduction

Since this article and ***Jordan derivation commuting with the projection on the smallest string of a triangle***, published in the same volume, are closely related, here we use the introduction and the preliminaries of this paper.

2 Jordan derivations commuting with derivation ∂_3

The projection on the greatest string of the triangle

$$\partial_3 : \triangle^{(n)}\{a,b,c\} \to \mathcal{STR}^{(n)}\{b,c\}$$

such that for any $\alpha = a_i b_j c_{n-i-j}$,

$$\partial_3(\alpha) = b_{i+j} c_{n-i-j} \in \mathcal{STR}^{(n)}\{b,c\}$$

ISSN 1311-0829

is a derivation – Theorem 5 of [10]. The maximal subsemiring $\mathcal{D}_c$ of the triangle closed under the derivation ∂_3 is consisting of all types except (a, c, c) and (b, c, c) – Theorem 6 of [10].

Theorem 16. *Let $\alpha \in \mathcal{D}_c$ and $\beta \in \mathcal{RI}\left(\triangle^{(n)}\{a, b, c\}\right)$. Then $\partial_3\partial_\beta(\alpha) = \partial_\beta\partial_3(\alpha)$.*

Proof. Let $\beta = a_k b_\ell c_{n-k-\ell}$, $a \leq k-1 < b \leq k+\ell-1 < c$ is an element of $\mathcal{RI}\left(\triangle^{(n)}\{a, b, c\}\right)$. Then for any α, it follows that Jordan multiplication $\partial_\beta(\alpha) = \alpha\beta + \beta\alpha = \alpha + \beta\alpha$ is a derivation, see [11].

Case 1. Let $\alpha \in (a, a, a)$. As in case 1 of Theorem 1 we find $\beta\alpha = \overline{a}$ and $\partial_\beta(\alpha) = \alpha$. Since $\partial_3(\alpha) \in (b, b, b)$, it follows $\beta\partial_3(\alpha) = \overline{b}$. We obtain $\partial_\beta\partial_3(\alpha) = \partial_3(\partial_\beta(\alpha)) = \partial_3(\alpha)$. Since $\partial_3(\alpha) > \overline{b}$, it follows

$$\partial_3\partial_\beta(\alpha) = \partial_\beta(\partial_3(\alpha)) = \beta\partial_3(\alpha) + \partial_3(\alpha) = \overline{b} + \partial_3(\alpha) = \partial_3(\alpha) = \partial_\beta\partial_3(\alpha).$$

Case 2. Let $\alpha \in (a, a, b)$. Then $\alpha = a_i b_j c_{n-i-j}$, where $a < b \leq i-1 < c \leq i+j-1$. As in case 2 of Theorem 1 we obtain $\beta\alpha = a_{k+\ell} b_{n-k-\ell}$ and then $\partial_\beta(\alpha) = \alpha + a_{k+\ell} b_{n-k-\ell}$. Since $\partial_3(\alpha) \in (b, b, b)$, it follows $\beta\partial_3(\alpha) = \overline{b}$. Using $\partial_3(\alpha) > \overline{b}$ we have $\partial_3\partial_\beta(\alpha) = \partial_\beta(\partial_3(\alpha)) = \partial_3(\alpha) + \beta\partial_3(\alpha) = \partial_3(\alpha)$. Hence,

$$\partial_\beta\partial_3(\alpha) = \partial_3(\partial_\beta(\alpha)) = \partial_3(\alpha + a_{k+\ell} b_{n-k-\ell}) = \partial_3(\alpha) + \overline{b} = \partial_3(\alpha) = \partial_3\partial_\beta(\alpha).$$

Case 3. Let $\alpha \in (a, b, b)$. Then $\alpha = a_i b_j c_{n-i-j}$, where $a \leq i-1 < b < c \leq i+j-1$. As in case 3 of Theorem 1 we obtain $\beta\alpha = a_k b_{n-k}$ and then $\partial_\beta(\alpha) = \alpha + a_k b_{n-k}$. Since $\partial_3(\alpha) \in (b, b, b)$, it follows $\beta\partial_3(\alpha) = \overline{b}$. But $\partial_3(\alpha) > \overline{b}$. Then we find

$$\partial_3\partial_\beta(\alpha) = \partial_\beta(\partial_3(\alpha)) = \partial_3(\alpha) + \beta\partial_3(\alpha) = \partial_3(\alpha) + \overline{b} = \partial_3(\alpha).$$

Hence $\partial_\beta\partial_3(\alpha) = \partial_3(\partial_\beta(\alpha)) = \partial_3(\alpha + a_k b_{n-k}) = \partial_3(\alpha) + \overline{b} = \partial_3(\alpha) = \partial_3\partial_\beta(\alpha)$.

Case 4. Let $\alpha \in (b, b, b)$. We obtain $\beta\alpha = \overline{b}$ and then $\partial_\beta(\alpha) = \alpha + \overline{b} = \alpha$. Since $\partial_3(\alpha) \in (b, b, b)$, it follows $\partial_3\partial_\beta(\alpha) = \partial_\beta(\partial_3(\alpha)) = \partial_3(\alpha) = \partial_3(\partial_\beta(\alpha)) = \partial_\beta\partial_3(\alpha)$.

Case 5. Let $\alpha \in (a, a, c)$. Then $\alpha = a_i b_j c_{n-i-j}$, where $a < b \leq i-1 < i+j \leq c$. We obtain $\beta\alpha = a_{k+\ell} c_{n-k-\ell}$ and then $\partial_\beta(\alpha) = \alpha + a_{k+\ell} c_{n-k-\ell}$. Since $\partial_3(\alpha) = b_{i+j} c_{n-i-j}$, it follows $\beta\partial_3(\alpha) = a_k b_\ell c_{n-k-\ell} \cdot b_{i+j} c_{n-i-j} = b_{k+\ell} c_{n-k-\ell}$. Thus, we have $\partial_3\partial_\beta(\alpha) = \partial_\beta(\partial_3(\alpha)) = \partial_3(\alpha) + \beta\partial_3(\alpha) = \partial_3(\alpha) + b_{k+\ell} c_{n-k-\ell}$. Hence, $\partial_\beta\partial_3(\alpha) = \partial_3(\partial_\beta(\alpha)) = \partial_3(\alpha + a_{k+\ell} c_{n-k-\ell}) = \partial_3(\alpha) + b_{k+\ell} c_{n-k-\ell} = \partial_3\partial_\beta(\alpha)$.

Case 6. Let $\alpha \in (a, b, c)$. Now $\alpha = a_i b_j c_{n-i-j}$, where $a \leq i-1 < b \leq i+j-1 < c$. Then α is a right identity and $\partial_\beta(\alpha) = \alpha + \beta$. So,

$$\partial_\beta\partial_3(\alpha) = \partial_3(\partial_\beta(\alpha)) = \partial_3(\alpha + \beta) = \partial_3(\alpha) + \partial_3(\beta) = \partial_3(\alpha) + b_{k+\ell} c_{n-k-\ell}.$$

Since $\partial_3(\alpha) = b_{i+j}c_{n-i-j}$, it follows $\beta\partial_1(\alpha) = a_k b_\ell c_{n-k-\ell} \cdot b_{i+j}c_{n-i-j} = b_{k+\ell}c_{n-k-\ell}$. Hence, $\partial_3\partial_\beta(\alpha) = \partial_\beta(\partial_3(\alpha)) = \partial_3(\alpha) + \beta\partial_3(\alpha) = \partial_3(\alpha) + b_{k+\ell}c_{n-k-\ell} = \partial_\beta\partial_3(\alpha)$.

Case 7. Let $\alpha \in (b, b, c)$. Then $\alpha = a_i b_j c_{n-i-j}$, where $i \leq a < b \leq i + j - 1 < c$. We obtain $\beta\alpha = a_k b_\ell c_{n-k-\ell} \cdot a_i b_j c_{n-i-j} = b_{k+\ell}c_{n-k-\ell}$ and then $\partial_\beta(\alpha) = \alpha + b_{k+\ell}c_{n-k-\ell}$. Since $\partial_3(\alpha) \in (b, b, c)$, it follows $\partial_3\partial_\beta(\alpha) = \partial_\beta(\partial_3(\alpha)) = \partial_3(\alpha) + b_{k+\ell}c_{n-k-\ell}$. Hence, $\partial_\beta\partial_3(\alpha) = \partial_3(\partial_\beta(\alpha)) = \partial_3(\alpha + b_{k+\ell}c_{n-k-\ell}) = \partial_3(\alpha) + \partial_3(b_{k+\ell}c_{n-k-\ell}) = \partial_3(\alpha) + b_{k+\ell}c_{n-k-\ell} = \partial_3\partial_\beta(\alpha)$.

Case 8. Let $\alpha \in (c, c, c)$. We obtain $\beta\alpha = \overline{c}$ and then $\partial_\beta(\alpha) = \alpha + \overline{c} = \overline{c}$. Since $\partial_3(\alpha) \in (c, c, c)$, it follows $\partial_3\partial_\beta(\alpha) = \partial_\beta(\partial_3(\alpha)) = \overline{c} = \partial_3(\overline{c}) = \partial_3(\partial_\beta(\alpha)) = \partial_\beta\partial_3(\alpha)$.

Theorem 17. *Let $\alpha \in \mathcal{D}_c$ and $\beta \in (a, a, a)$. Then $\partial_3\partial_\beta(\alpha) = \partial_\beta\partial_3(\alpha)$.*

Proof. If $\beta \in (a, a, a)$ for any α we find $\partial_\beta(\alpha) = \alpha\beta + \beta\alpha = \overline{a} + \beta\alpha = \beta\alpha$.

Case 1. Let $\alpha \in (a, a, a)$ or $\alpha \in (a, a, b)$, or $\alpha \in (a, b, b)$. Then $\beta\alpha = \overline{a}$ and $\partial_\beta(\alpha) = \overline{a}$. Thus $\partial_\beta\partial_3(\alpha) = \partial_3(\partial_\beta(\alpha)) = \partial_3(\overline{a}) = \overline{b}$. Since $\partial_3(\alpha) \in (b, b, b)$, it follows $\beta\partial_3(\alpha) = \overline{b}$. Hence $\partial_3\partial_\beta(\alpha) = \partial_\beta(\partial_3(\alpha)) = \beta\partial_3(\alpha) = \overline{b} = \partial_\beta\partial_3(\alpha)$.

Case 2. Let $\alpha \in (b, b, b)$. Then $\beta\alpha = \overline{b}$ and $\partial_\beta(\alpha) = \overline{b}$. Since $\partial_3(\alpha) \in (b, b, b)$, it follows $\partial_3\partial_\beta(\alpha) = \partial_\beta(\partial_3(\alpha)) = \overline{b}$. Hence $\partial_\beta\partial_3(\alpha) = \partial_3(\partial_\beta(\alpha)) = \partial_3(\overline{b}) = \overline{b} = \partial_3\partial_\beta(\alpha)$.

Case 3. Let $\alpha \in (a, a, c)$ or $\alpha \in (a, b, c)$. Then $\beta\alpha = \overline{a}$ and $\partial_\beta(\alpha) = \overline{a}$. As in case 1 we find $\partial_\beta\partial_3(\alpha) = \partial_3(\partial_\beta(\alpha)) = \partial_3(\overline{a}) = \overline{b}$. Since $\partial_3(\alpha) \in (b, b, c)$, it follows $\beta\partial_3(\alpha) = \overline{b}$. Hence $\partial_3\partial_\beta(\alpha) = \partial_\beta(\partial_3(\alpha)) = \beta\partial_3(\alpha) = \overline{b} = \partial_\beta\partial_3(\alpha)$.

Case 4. Let $\alpha \in (b, b, c)$. Then $\beta\alpha = \overline{b}$ and $\partial_\beta(\alpha) = \overline{b}$. As in case 2 we find $\partial_\beta\partial_3(\alpha) = \partial_3(\partial_\beta(\alpha)) = \partial_3(\overline{b}) = \overline{b}$. Since $\partial_3(\alpha) \in (b, b, c)$, it follows

$$\partial_3\partial_\beta(\alpha) = \partial_\beta(\partial_3(\alpha)) = \overline{b} = \partial_\beta\partial_3(\alpha).$$

Case 5. Let $\alpha \in (c, c, c)$. Then $\beta\alpha = \overline{c}$ and $\partial_\beta(\alpha) = \overline{c}$. Since $\partial_3(\alpha) \in (c, c, c)$, it follows $\partial_\beta(\partial_3(\alpha)) = \overline{c}$. Hence $\partial_3\partial_\beta(\alpha) = \partial_\beta(\partial_3(\alpha)) = \overline{c} = \partial_3(\overline{c}) = \partial_3(\partial_\beta(\alpha)) = \partial_\beta\partial_3(\alpha)$.

Theorem 18. *Let $\alpha \in \mathcal{D}_c$ and $\beta \in (b, b, b)$. Then $\partial_3\partial_\beta(\alpha) = \partial_\beta\partial_3(\alpha)$.*

Proof. If $\beta \in (b, b, b)$ for any α we find $\partial_\beta(\alpha) = \alpha\beta + \beta\alpha = \overline{b} + \beta\alpha$.

Case 1. Let $\alpha(b) \leq b$. Then $\beta\alpha = \overline{a}$ or $\beta\alpha = \overline{b}$. So, $\partial_\beta(\alpha) = \overline{b}$. We obtain $\partial_\beta\partial_3(\alpha) = \partial_3(\partial_\beta(\alpha)) = \partial_3(\overline{b}) = \overline{b}$. Since $\partial_3(\alpha) \in (b, b, b)$ or $\partial_3(\alpha) \in (b, b, c)$, it follows $\beta\partial_3(\alpha) = \overline{b}$. Hence $\partial_3\partial_\beta(\alpha) = \partial_\beta(\partial_3(\alpha)) = \beta\partial_3(\alpha) + \partial_3(\alpha)\beta = \overline{b} = \partial_\beta\partial_3(\alpha)$.

Case 2. Let $\alpha \in (c, c, c)$. Then $\beta\alpha = \overline{c}$ and $\partial_\beta(\alpha) = \overline{c}$. We obtain $\partial_\beta\partial_3(\alpha) = \partial_3(\partial_\beta(\alpha)) = \partial_3(\overline{c}) = \overline{c}$. Since $\partial_3(\alpha) \in (c, c, c)$, it follows $\partial_\beta(\partial_3(\alpha)) = \overline{c}$. Hence

$$\partial_3\partial_\beta(\alpha) = \partial_\beta(\partial_3(\alpha)) = \overline{c} = \partial_\beta\partial_3(\alpha).$$

Theorem 19. *Let $\alpha \in \mathcal{D}_c$ and $\beta \in (c,c,c)$. Then $\partial_3\partial_\beta(\alpha) = \partial_\beta\partial_3(\alpha)$.*

Proof. If $\beta \in (c,c,c)$ for any α we find $\partial_\beta(\alpha) = \alpha\beta + \beta\alpha = \overline{c} + \beta\alpha = \overline{c}$.

Then we obtain $\partial_\beta\partial_3(\alpha) = \partial_3(\partial_\beta(\alpha)) = \partial_3(\overline{c}) = \overline{c}$. Hence

$$\partial_3\partial_\beta(\alpha) = \partial_\beta(\partial_3(\alpha)) = \beta\partial_3(\alpha) + \partial_3(\alpha)\beta = \beta\partial_3(\alpha) + \overline{c} = \overline{c} = \partial_\beta\partial_3(\alpha).$$

Theorem 20. *Let $\alpha \in \mathcal{D}_c$ and $\beta \in (b,b,c)$. Then $\partial_3\partial_\beta(\alpha) = \partial_\beta\partial_3(\alpha)$.*

Proof. Let $\beta = a_k b_\ell c_{n-k-\ell}$, where $k \leq a < b < k+\ell \leq c$.

Case 1. Let $\alpha(c) \leq b$, i.e. $\alpha \in (a,a,a)$ or $\alpha \in (a,a,b)$, or $\alpha \in (a,b,b)$, or $\alpha \in (b,b,b)$. Then $\alpha\beta = \overline{b}$ and $\beta\alpha = \overline{a}$ or $\beta\alpha = \overline{b}$. So, a$\partial_\beta(\alpha) = \overline{b}$. Thus $\partial_\beta\partial_3(\alpha) = \partial_3(\partial_\beta(\alpha)) = \partial_3(\overline{b}) = \overline{b}$. Since $\partial_3(\alpha) \in (b,b,b)$, it follows $\beta\partial_3(\alpha) = \overline{b}$. Hence

$$\partial_3\partial_\beta(\alpha) = \partial_\beta(\partial_3(\alpha)) = \beta\partial_3(\alpha) + \partial_3(\alpha)\beta = \overline{b} + \overline{b} = \overline{b} = \partial_\beta\partial_3(\alpha).$$

Case 2. Let $\alpha \in (a,a,c)$. Then $\alpha = a_i b_j c_{n-i-j}$, where $a < b \leq i-1 < i+j \leq c$.

Now we obtain

$$\partial_\beta(\alpha) = \alpha\beta + \beta\alpha = a_i b_j c_{n-i-j} \cdot a_k b_\ell c_{n-k-\ell} + a_k b_\ell c_{n-k-\ell} \cdot a_i b_j c_{n-i-j} = b_{i+j} c_{n-i-j} + a_{k+\ell} c_{n-k-\ell}.$$

Thus $\partial_\beta\partial_3(\alpha) = \partial_3(\partial_\beta(\alpha)) = \partial_3(b_{i+j}c_{n-i-j} + a_{k+\ell}c_{n-k-\ell}) =$

$$= \partial_3(b_{i+j}c_{n-i-j}) + \partial_3(a_{k+\ell}c_{n-k-\ell}) = b_{i+j}c_{n-i-j} + b_{k+\ell}c_{n-k-\ell}.$$

Since $\partial_3(\alpha) = b_{i+j}c_{n-i-j}$, it follows

$$\partial_3\partial_\beta(\alpha) = \partial_\beta(\partial_3(\alpha)) = \beta\partial_3(\alpha) + \partial_3(\alpha)\beta = a_k b_\ell c_{n-k-\ell} \cdot b_{i+j} c_{n-i-j} + b_{i+j} c_{n-i-j} \cdot a_k b_\ell c_{n-k-\ell} =$$

$$= b_{k+\ell}c_{n-k-\ell} + b_{i+j}c_{n-i-j} = \partial_\beta\partial_3(\alpha).$$

Case 3. Let $\alpha \in (a,b,c)$. Then $\alpha = a_i b_j c_{n-i-j}$, where $a < i \leq b < i+j \leq c$. Since α is a right identity, it follows $\partial_\beta(\alpha) = \alpha\beta + \beta$. But $\alpha\beta = a_i b_j c_{n-i-j} \cdot a_k b_\ell c_{n-k-\ell} = b_{i+j}c_{n-i-j}$. So, $\partial_\beta(\alpha) = b_{i+j}c_{n-i-j} + a_k b_\ell c_{n-k-\ell}$.

Thus $\partial_\beta\partial_3(\alpha) = \partial_3(\partial_\beta(\alpha)) = \partial_3(b_{i+j}c_{n-i-j} + a_k b_\ell c_{n-k-\ell}) =$

$$= \partial_3(b_{i+j}c_{n-i-j}) + \partial_3(a_k b_\ell c_{n-k-\ell}) = b_{i+j}c_{n-i-j} + b_{k+\ell}c_{n-k-\ell}.$$

As in the previous case, it follows $\partial_3\partial_\beta(\alpha) = \partial_\beta(\partial_3(\alpha)) = \beta\partial_3(\alpha) + \partial_3(\alpha)\beta =$

$$= a_k b_\ell c_{n-k-\ell} \cdot b_{i+j} c_{n-i-j} + b_{i+j} c_{n-i-j} \cdot a_k b_\ell c_{n-k-\ell} = b_{k+\ell}c_{n-k-\ell} + b_{i+j}c_{n-i-j} = \partial_\beta\partial_3(\alpha).$$

Case 4. Let $\alpha \in (b,b,c)$. Similarly of the two previous cases we find $\partial_\beta(\alpha) = b_{i+j}c_{n-i-j} + b_{k+\ell}c_{n-k-\ell}$ and then $\partial_\beta\partial_3(\alpha) = \partial_3(\partial_\beta(\alpha)) = \partial_3(b_{i+j}c_{n-i-j} + b_{k+\ell}c_{n-k-\ell}) =$

$$= \partial_3(b_{i+j}c_{n-i-j}) + \partial_3(b_{k+\ell}c_{n-k-\ell}) = b_{i+j}c_{n-i-j} + b_{k+\ell}c_{n-k-\ell}.$$

As in the previous case, it follows $\partial_3\partial_\beta(\alpha) = \partial_\beta(\partial_3(\alpha)) = \beta\partial_3(\alpha) + \partial_3(\alpha)\beta =$

$$= a_k b_\ell c_{n-k-\ell} \cdot b_{i+j} c_{n-i-j} + b_{i+j} c_{n-i-j} \cdot a_k b_\ell c_{n-k-\ell} =$$

$$= b_{k+\ell} c_{n-k-\ell} + b_{i+j} c_{n-i-j} = \partial_\beta\partial_3(\alpha).$$

Case 5. Let $\alpha \in (c, c, c)$. As in case 2 of Theorem 18 we find $\beta\alpha = \overline{c}$ and then $\partial_\beta(\alpha) = \overline{c}$. We obtain $\partial_\beta\partial_3(\alpha) = \partial_3(\partial_\beta(\alpha)) = \partial_3(\overline{c}) = \overline{c}$. Since $\partial_3(\alpha) \in (c, c, c)$, it follows $\partial_\beta(\partial_3(\alpha)) = \overline{c}$. Hence

$$\partial_3\partial_\beta(\alpha) = \partial_\beta(\partial_3(\alpha)) = \overline{c} = \partial_\beta\partial_3(\alpha).$$

Not all Jordan derivations commutes with the derivation ∂_3 as we now show.

Proposition 9. *Let $\alpha \in (a, a, c)$ and $\beta \in (a, b, b)$. Then $\partial_3\partial_\beta(\alpha) \neq \partial_\beta\partial_3(\alpha)$.*

Proof. Since $\alpha \in (a, a, c)$ we can write $\alpha = a_i b_j c_{n-i-j}$, where $a < b \leq i - 1 < i + j \leq c$. Since $\beta \in (a, b, b)$ we can write $\beta = a_k b_\ell c_{n-k-\ell}$, where $a \leq k - 1 < b < c \leq k + \ell - 1$.

Now we obtain

$$\partial_\beta(\alpha) = a_i b_j c_{n-i-j} \cdot a_k b_\ell c_{n-k-\ell} + a_k b_\ell c_{n-k-\ell} \cdot a_i b_j c_{n-i-j} = a_i b_{n-i} + a_{k+\ell} c_{n-k-\ell}.$$

Thus $\partial_\beta\partial_3(\alpha) = \partial_3(\partial_\beta(\alpha)) = \partial_3(a_i b_{n-i} + a_{k+\ell} c_{n-k-\ell}) =$

$$= \partial_3(a_i b_{n-i}) + \partial_3(a_{k+\ell} c_{n-k-\ell}) = \overline{b} + b_{k+\ell} c_{n-k-\ell} = b_{k+\ell} c_{n-k-\ell}.$$

Since $\partial_3(\alpha) = b_{i+j} c_{n-i-j}$, it follows $\partial_3\partial_\beta(\alpha) = \partial_\beta(\partial_3(\alpha)) = \beta\partial_3(\alpha) + \partial_3(\alpha)\beta =$

$$= a_k b_\ell c_{n-k-\ell} \cdot b_{i+j} c_{n-i-j} + b_{i+j} c_{n-i-j} \cdot a_k b_\ell c_{n-k-\ell} = b_{k+\ell} c_{n-k-\ell} + b_{i+j} c_{n-i-j}.$$

But $i + j \leq c < k + \ell$, so, $b_{k+\ell} c_{n-k-\ell} < b_{i+j} c_{n-i-j}$. Hence

$$\partial_3\partial_\beta(\alpha) = b_{i+j} c_{n-i-j} > b_{k+\ell} c_{n-k-\ell} = \partial_\beta\partial_3(\alpha).$$

Theorem 21. *Let $\alpha \in \mathcal{D}_c$, $\alpha \notin (a, a, c)$ and $\beta \in (a, b, b)$. Then $\partial_3\partial_\beta(\alpha) = \partial_\beta\partial_3(\alpha)$.*

Proof. Let $\beta = a_k b_\ell c_{n-k-\ell}$, where $a \leq k - 1 < b < c \leq k + \ell - 1$.

Case 1. Let $\alpha \in (a, a, a)$. Now $\alpha = a_i b_j c_{n-i-j}$, where $a < b < c \leq i - 1$. We find

$$\partial_\beta(\alpha) = a_i b_j c_{n-i-j} \cdot a_k b_\ell c_{n-k-\ell} + a_k b_\ell c_{n-k-\ell} \cdot a_i b_j c_{n-i-j} = a_i b_{n-i} + \overline{a} = a_i b_{n-i}.$$

We obtain

$$\partial_\beta\partial_3(\alpha) = \partial_3(\partial_\beta(\alpha)) = \partial_3(a_i b_{n-i}) = \overline{b}.$$

Since $\partial_3(\alpha) \in (b,b,b)$, it follows $\beta\partial_3(\alpha) = \bar{b}$ and $\partial_3(\alpha)\beta = \bar{b}$. Hence

$$\partial_3\partial_\beta(\alpha) = \partial_\beta(\partial_3(\alpha)) = \beta\partial_3(\alpha) + \partial_3(\alpha)\beta = \bar{b} + \bar{b} = \bar{b} = \partial_\beta\partial_3(\alpha).$$

Case 2. Let $\alpha \in (a,a,b)$. Now $\alpha = a_i b_j c_{n-i-j}$, where $a < b \leq i-1 < c \leq i+j-1$ and then $\partial_\beta(\alpha) = \alpha\beta + \beta\alpha = a_i b_j c_{n-i-j} \cdot a_k b_\ell c_{n-k-\ell} + a_k b_\ell c_{n-k-\ell} \cdot a_i b_j c_{n-i-j} = a_i b_{n-i} + a_{k+\ell} b_{n-k-\ell}$. Since $i \leq c \leq k+\ell$ we find $a_{k+\ell} b_{n-k-\ell} \leq a_i b_{n-i}$ and $\partial_\beta(\alpha) = a_i b_{n-i}$. Thus

$$\partial_\beta\partial_3(\alpha) = \partial_3(\partial_\beta(\alpha)) = \partial_3(a_i b_{n-i}) = \bar{b}.$$

As in the previous case since $\partial_3(\alpha) \in (b,b,b)$, it follows $\beta\partial_3(\alpha) = \bar{b}$ and $\partial_3(\alpha)\beta = \bar{b}$. Hence

$$\partial_3\partial_\beta(\alpha) = \partial_\beta(\partial_3(\alpha)) = \beta\partial_3(\alpha) + \partial_3(\alpha)\beta = \bar{b} + \bar{b} = \bar{b} = \partial_\beta\partial_3(\alpha).$$

Case 3. Let $\alpha \in (a,b,b)$. Now $\alpha = a_i b_j c_{n-i-j}$, where $a \leq i-1 < b < c \leq i+j-1$ and then $\partial_\beta(\alpha) = \alpha\beta + \beta\alpha = a_i b_j c_{n-i-j} \cdot a_k b_\ell c_{n-k-\ell} + a_k b_\ell c_{n-k-\ell} \cdot a_i b_j c_{n-i-j} = a_i b_{n-i} + a_k b_{n-k}$. Now we find

$$\partial_\beta\partial_3(\alpha) = \partial_3(\partial_\beta(\alpha)) = \partial_3(a_i b_{n-i} + a_k b_{n-k}) = \partial_3(a_i b_{n-i}) + \partial_3(a_k b_{n-k}) = \bar{b} + \bar{b} = \bar{b}.$$

As in the previous two cases since $\partial_3(\alpha) \in (b,b,b)$, it follows $\beta\partial_3(\alpha) = \bar{b}$ and $\partial_3(\alpha)\beta = \bar{b}$. Hence

$$\partial_3\partial_\beta(\alpha) = \partial_\beta(\partial_3(\alpha)) = \beta\partial_3(\alpha) + \partial_3(\alpha)\beta = \bar{b} + \bar{b} = \bar{b} = \partial_\beta\partial_3(\alpha).$$

Case 4. Let $\alpha \in (b,b,b)$. Then $\alpha\beta = \bar{b}$, $\beta\alpha = \bar{b}$ and $\partial_\beta(\alpha) = \bar{b}$. We obtain

$$\partial_\beta\partial_3(\alpha) = \partial_3(\partial_\beta(\alpha)) = \partial_3(\bar{b}) = \bar{b}.$$

As in the case 1 we have $\partial_3(\alpha) \in (b,b,b)$ and then $\beta\partial_3(\alpha) = \bar{b}$ and $\partial_3(\alpha)\beta = \bar{b}$. Hence

$$\partial_3\partial_\beta(\alpha) = \partial_\beta(\partial_3(\alpha)) = \beta\partial_3(\alpha) + \partial_3(\alpha)\beta = \bar{b} + \bar{b} = \bar{b} = \partial_\beta\partial_3(\alpha).$$

Case 5. Let $\alpha \in (a,b,c)$. Then $\alpha = a_i b_j c_{n-i-j}$, where $a < i \leq b < i+j \leq c$. Now α is a right identity and $\partial_\beta(\alpha) = \alpha\beta + \beta$.

But $\alpha\beta = a_i b_j c_{n-i-j} \cdot a_k b_\ell c_{n-k-\ell} = a_i b_{n-i}$. So, $\partial_\beta(\alpha) = a_i b_{n-i} + a_k b_\ell c_{n-k-\ell}$.

Thus $\partial_\beta\partial_3(\alpha) = \partial_3(\partial_\beta(\alpha)) = \partial_3(a_i b_{n-i} + a_k b_\ell c_{n-k-\ell}) =$

$$= \partial_3(a_i b_{n-i}) + \partial_3(a_k b_\ell c_{n-k-\ell}) = \bar{b} + b_{k+\ell} c_{n-k-\ell} = b_{k+\ell} c_{n-k-\ell}.$$

As in the previous case, it follows

$$\partial_3\partial_\beta(\alpha) = \partial_\beta(\partial_3(\alpha)) = \beta\partial_3(\alpha) + \partial_3(\alpha)\beta = a_k b_\ell c_{n-k-\ell} \cdot b_{i+j} c_{n-i-j} + b_{i+j} c_{n-i-j} \cdot a_k b_\ell c_{n-k-\ell} =$$

$$= b_{k+\ell}c_{n-k-\ell} + \overline{b} = b_{k+\ell}c_{n-k-\ell} = \partial_\beta\partial_3(\alpha).$$

Case 6. Let $\alpha \in (b, b, c)$. Then $\alpha = a_i b_j c_{n-i-j}$, where $i \le a < b \le i + j - 1 < c$. We obtain $\partial_\beta(\alpha) = \alpha\beta + \beta\alpha = a_i b_{n-i} + b_{k+\ell}c_{n-k-\ell}$. Thus

$$\partial_\beta\partial_3(\alpha) = \partial_3(\partial_\beta(\alpha)) = \partial_3(a_i b_{n-i} + b_{k+\ell}c_{n-k-\ell}) =$$

$$= \partial_3(a_i b_{n-i}) + \partial_3(b_{k+\ell}c_{n-k-\ell}) = \overline{b} + b_{k+\ell}c_{n-k-\ell} = b_{k+\ell}c_{n-k-\ell}.$$

Since $\partial_3(\alpha) = b_{i+j}c_{n-i-j}$, it follows $\partial_3\partial_\beta(\alpha) = \partial_\beta(\partial_3(\alpha)) = \partial_3(\alpha)\beta + \beta\partial_3(\alpha) =$

$$= b_{i+j}c_{n-i-j} \cdot a_k b_\ell c_{n-k-\ell} + a_k b_\ell c_{n-k-\ell} \cdot b_{i+j}c_{n-i-j} = \overline{b} + b_{k+\ell}c_{n-k-\ell} = b_{k+\ell}c_{n-k-\ell} = \partial_\beta\partial_3(\alpha).$$

Case 7. Let $\alpha \in (c, c, c)$. We obtain $\beta\alpha = \overline{c}$ and then $\partial_\beta(\alpha) = \alpha\beta + \overline{c} = \overline{c}$. Since $\partial_3(\alpha) \in (c, c, c)$, it follows

$$\partial_3\partial_\beta(\alpha) = \partial_\beta(\partial_3(\alpha)) = \overline{c} = \partial_3(\overline{c}) = \partial_3(\partial_\beta(\alpha)) = \partial_\beta\partial_3(\alpha).$$

Proposition 10. *Let $\alpha \in (a, a, c)$ and $\beta \in (a, a, b)$. Then $\partial_3\partial_\beta(\alpha) \neq \partial_\beta\partial_3(\alpha)$.*

Proof. As in the proof of Proposition 3 we find $\partial_3\partial_\beta(\alpha) < \partial_\beta\partial_3(\alpha)$.

Theorem 22. *Let $\alpha \in \mathcal{D}_c$, $\alpha \notin (a, a, c)$ and $\beta \in (a, a, b)$. Then $\partial_3\partial_\beta(\alpha) = \partial_\beta\partial_3(\alpha)$.*

Proof. Let $\beta = a_k b_\ell c_{n-k-\ell}$, where $a < b \le k - 1 < c \le k + \ell - 1$.

Case 1. Let $\alpha \in (a, a, a)$ or $\alpha \in (a, a, b)$. Then, as in case 1 of Theorem 14, we find $\alpha\beta = \overline{a}$, $\beta\alpha = \overline{a}$ and $\partial_\beta(\alpha) = \overline{a}$. We obtain $\partial_\beta\partial_3(\alpha) = \partial_3(\partial_\beta(\alpha)) = \partial_3(\overline{a}) = \overline{b}$. Since $\partial_3(\alpha) \in (b, b, b)$, it follows $\beta\partial_3(\alpha) = \overline{b}$ and $\partial_3(\alpha)\beta = \overline{a}$. Hence

$$\partial_3\partial_\beta(\alpha) = \partial_\beta(\partial_3(\alpha)) = \beta\partial_3(\alpha) + \partial_3(\alpha)\beta = \overline{b} + \overline{a} = \overline{b} = \partial_\beta\partial_3(\alpha).$$

Case 2. Let $\alpha \in (a, b, b)$. Now $\alpha = a_i b_j c_{n-i-j}$, where $a \le i-1 < b < c \le i+j-1$ and then $\partial_\beta(\alpha) = \alpha\beta + \beta\alpha = a_i b_j c_{n-i-j} \cdot a_k b_\ell c_{n-k-\ell} + a_k b_\ell c_{n-k-\ell} \cdot a_i b_j c_{n-i-j} = \overline{a} + a_k b_{n-k} = a_k b_{n-k}$. Now we find $\partial_\beta\partial_3(\alpha) = \partial_3(\partial_\beta(\alpha)) = \partial_3(a_k b_{n-k}) = \overline{b}$. Since $\partial_3(\alpha) \in (b, b, b)$, it follows $\beta\partial_3(\alpha) = \overline{b}$ and $\partial_3(\alpha)\beta = \overline{a}$. Hence

$$\partial_3\partial_\beta(\alpha) = \partial_\beta(\partial_3(\alpha)) = \beta\partial_3(\alpha) + \partial_3(\alpha)\beta = \overline{b} + \overline{a} = \overline{b} = \partial_\beta\partial_3(\alpha).$$

Case 3. Let $\alpha \in (b, b, b)$. Then $\alpha\beta = \overline{a}$, $\beta\alpha = \overline{b}$ and $\partial_\beta(\alpha) = \overline{b}$. We obtain $\partial_\beta\partial_3(\alpha) = \partial_3(\partial_\beta(\alpha)) = \partial_3(\overline{b}) = \overline{b}$. Since $\partial_3(\alpha) \in (b, b, b)$, it follows $\beta\partial_3(\alpha) = \overline{b}$ and $\partial_3(\alpha)\beta = \overline{a}$. Hence $\partial_3\partial_\beta(\alpha) = \partial_\beta(\partial_3(\alpha)) = \beta\partial_3(\alpha) + \partial_3(\alpha)\beta = \overline{b} + \overline{a} = \overline{b} = \partial_\beta\partial_3(\alpha)$.

Case 4. Let $\alpha \in (a, b, c)$. Then $\alpha = a_i b_j c_{n-i-j}$, where $a < i \le b < i + j \le c$. Now α is a right identity and $\partial_\beta(\alpha) = \alpha\beta + \beta$. But $\alpha\beta = a_i b_j c_{n-i-j} \cdot a_k b_\ell c_{n-k-\ell} = a_{i+j} b_{n-i-j}$. So, $\partial_\beta(\alpha) = a_{i+j} b_{n-i-j} + a_k b_\ell c_{n-k-\ell}$. Thus $\partial_\beta\partial_3(\alpha) = \partial_3(\partial_\beta(\alpha)) =$

$$= \partial_3(a_{i+j} b_{n-i-j} + a_k b_\ell c_{n-k-\ell}) = \overline{b} + b_{k+\ell}c_{n-k-\ell} = b_{k+\ell}c_{n-k-\ell}.$$

Since $\partial_3(\alpha) = b_{i+j}c_{n-i-j}$, it follows $\partial_3\partial_\beta(\alpha) = \partial_\beta(\partial_3(\alpha)) = \beta\partial_3(\alpha) + \partial_3(\alpha)\beta =$

$$= b_{k+\ell}c_{n-k-\ell} + a_{i+j}b_{n-i-j} = b_{k+\ell}c_{n-k-\ell} = \partial_\beta\partial_3(\alpha).$$

Case 5. Let $\alpha \in (b,b,c)$. Then $\alpha = a_i b_j c_{n-i-j}$, where $i \leq a < b \leq i+j-1 < c$. We obtain $\partial_\beta(\alpha) = \alpha\beta + \beta\alpha = a_{i+j}b_{n-i-j} + b_{k+\ell}c_{n-k-\ell}$. Thus

$$\partial_\beta\partial_3(\alpha) = \partial_3(\partial_\beta(\alpha)) = \partial_3(a_{i+j}b_{n-i-j} + b_{k+\ell}c_{n-k-\ell}) =$$

$$= \partial_3(a_{i+j}b_{n-i-j}) + \partial_3(b_{k+\ell}c_{n-k-\ell}) = \overline{b} + b_{k+\ell}c_{n-k-\ell} = b_{k+\ell}c_{n-k-\ell}.$$

Since $\partial_3(\alpha) = b_{i+j}c_{n-i-j}$, it follows $\partial_3\partial_\beta(\alpha) = \partial_\beta(\partial_3(\alpha)) = \partial_3(\alpha)\beta + \beta\partial_3(\alpha) =$

$$= b_{i+j}c_{n-i-j} \cdot a_k b_\ell c_{n-k-\ell} + a_k b_\ell c_{n-k-\ell} \cdot b_{i+j}c_{n-i-j} =$$

$$= a_{i+j}b_{n-i-j} + b_{k+\ell}c_{n-k-\ell} = b_{k+\ell}c_{n-k-\ell} = \partial_\beta\partial_3(\alpha).$$

Case 6. Let $\alpha \in (c,c,c)$. As in case 7 of the previous theorem $\beta\alpha = \overline{c}$ and then $\partial_\beta(\alpha) = \alpha\beta + \overline{c} = \overline{c}$. Since $\partial_3(\alpha) \in (c,c,c)$, it follows

$$\partial_3\partial_\beta(\alpha) = \partial_\beta(\partial_3(\alpha)) = \beta\partial_3(\alpha) + \partial_3(\alpha)\beta = \overline{c} = \partial_3(\overline{c}) = \partial_3(\partial_\beta(\alpha)) = \partial_\beta\partial_3(\alpha).$$

Theorem 23. *Let $\alpha \in \mathcal{D}_c$ and $\beta \in (a,a,c)$. Then $\partial_3\partial_\beta(\alpha) = \partial_\beta\partial_3(\alpha)$.*

Proof. If $\beta \in (a,a,c)$, then $\beta = a_k b_\ell c_{n-k-\ell}$, where $a < b \leq k-1 < k+\ell \leq c$.

Case 1. Let $\alpha(c) \leq b$, i.e. $\alpha \in (a,a,a)$ or $\alpha \in (a,a,b)$, or $\alpha \in (a,b,b)$, or $\alpha \in (b,b,b)$. Then $\alpha\beta = \overline{a}$. So, $\partial_\beta(\alpha) = \overline{a} + \beta\alpha = \beta\alpha$. Since $\partial_3(\alpha) \in (b,b,b)$, it follows $\beta\partial_3(\alpha) = \overline{b}$ and $\partial_3(\alpha)\beta = \overline{a}$.

Thus $\partial_3\partial_\beta(\alpha) = \partial_\beta(\partial_3(\alpha)) = \beta\partial_3(\alpha) + \partial_3(\alpha)\beta = \overline{b} + \overline{a} = \overline{b}$. We obtain $\partial_\beta\partial_3(\alpha) = \partial_3(\partial_\beta(\alpha)) = \partial_3(\beta\alpha) = \partial_3(\beta)\alpha + \beta\partial_3(\alpha) = \partial_3(\beta)\alpha + \overline{b}$.

Since $\partial_3(\beta) = b_{k+\ell}c_{n-k-\ell}$, it follows $\partial_3(\beta)\alpha = b_{k+\ell}c_{n-k-\ell} \cdot \alpha$. But $b_{k+\ell}c_{n-k-\ell} \cdot \alpha(x) \leq b$ for $x = a,b,c$. So, $\partial_3(\beta)\alpha \leq \overline{b}$. Hence

$$\partial_\beta\partial_3(\alpha) = \partial_3(\beta)\alpha + \overline{b} = \overline{b} = \partial_3\partial_\beta(\alpha).$$

Case 2. Let $\alpha \in (c,c,c)$. As in the last case the previous theorem $\beta\alpha = \overline{c}$ and then $\partial_\beta(\alpha) = \alpha\beta + \overline{c} = \overline{c}$. Since $\partial_3(\alpha) \in (c,c,c)$, it follows

$$\partial_3\partial_\beta(\alpha) = \partial_\beta(\partial_3(\alpha)) = \beta\partial_3(\alpha) + \partial_3(\alpha)\beta = \overline{c} = \partial_3(\overline{c}) = \partial_3(\partial_\beta(\alpha)) = \partial_\beta\partial_3(\alpha).$$

Case 3. Let $\alpha \in (a,a,c)$. Then $\alpha = a_i b_j c_{n-i-j}$, where $a < b \leq i-1 < i+j \leq c$ and we find $\partial_\beta(\alpha) = \alpha\beta + \beta\alpha =$

$$= a_i b_j c_{n-i-j} \cdot a_k b_\ell c_{n-k-\ell} + a_k b_\ell c_{n-k-\ell} \cdot a_i b_j c_{n-i-j} = a_{i+j}c_{n-i-j} + a_{k+\ell}c_{n-k-\ell}.$$

We obtain $\partial_\beta\partial_3(\alpha) = \partial_3(\partial_\beta(\alpha)) = \partial_3(a_{i+j}c_{n-i-j} + a_{k+\ell}c_{n-k-\ell}) =$

$$= \partial_3(a_{i+j}c_{n-i-j}) + \partial_3(a_{k+\ell}c_{n-k-\ell}) = b_{i+j}c_{n-i-j} + b_{k+\ell}c_{n-k-\ell}.$$

Case 4. Let $\alpha \in (a, b, c)$. Then $\alpha = a_i b_j c_{n-i-j}$, where $a \leq i - 1 < b < i + j \leq c$ and since α is a right identity we find $\partial_\beta(\alpha) = \alpha\beta + \beta =$

$$= a_i b_j c_{n-i-j} \cdot a_k b_\ell c_{n-k-\ell} + a_k b_\ell c_{n-k-\ell} = a_{i+j}c_{n-i-j} + a_k b_\ell c_{n-k-\ell}.$$

We obtain $\partial_\beta\partial_3(\alpha) = \partial_3(\partial_\beta(\alpha)) = \partial_3(a_{i+j}c_{n-i-j} + a_k b_\ell c_{n-k-\ell}) =$

$$= \partial_3(a_{i+j}c_{n-i-j}) + \partial_3(a_k b_\ell c_{n-k-\ell}) = b_{i+j}c_{n-i-j} + b_{k+\ell}c_{n-k-\ell}.$$

Case 5. Let $\alpha \in (b, b, c)$. Then $\alpha = a_i b_j c_{n-i-j}$, where $i \leq a < b < i + j \leq c$ and we obtain $\partial_\beta(\alpha) = \alpha\beta + \beta =$

$$= a_i b_j c_{n-i-j} \cdot a_k b_\ell c_{n-k-\ell} + a_k b_\ell c_{n-k-\ell} = a_{i+j}c_{n-i-j} + b_{k+\ell}c_{n-k-\ell}.$$

We obtain $\partial_\beta\partial_3(\alpha) = \partial_3(\partial_\beta(\alpha)) = \partial_3(a_{i+j}c_{n-i-j} + b_{k+\ell}c_{n-k-\ell}) =$

$$= \partial_3(a_{i+j}c_{n-i-j}) + \partial_3(b_{k+\ell}c_{n-k-\ell}) = b_{i+j}c_{n-i-j} + b_{k+\ell}c_{n-k-\ell}.$$

In all of these three cases from $\partial_3(\alpha) = b_{i+j}c_{n-i-j}$, it follows

$$\partial_3\partial_\beta(\alpha) = \partial_\beta(\partial_3(\alpha)) = \partial_3(\alpha)\beta + \beta\partial_3(\alpha) =$$

$$= b_{i+j}c_{n-i-j} \cdot a_k b_\ell c_{n-k-\ell} + a_k b_\ell c_{n-k-\ell} \cdot b_{i+j}c_{n-i-j} = a_{i+j}c_{n-i-j} + b_{k+\ell}c_{n-k-\ell}.$$

If $k + \ell \leq i + j$ or $n - k - \ell \geq n - i - j$ we have

$$\partial_\beta\partial_3(\alpha) = b_{k+\ell}c_{n-k-\ell} = \partial_3\partial_\beta(\alpha).$$

If $k + \ell > i + j$ or $n - k - \ell < n - i - j$ we find

$$\partial_\beta\partial_3(\alpha) = b_{i+j}c_{n-i-j} = a_{i+j}c_{n-i-j} + b_{k+\ell}c_{n-k-\ell} = \partial_3\partial_\beta(\alpha)$$

and this completes the proof.

3 Conclusions

Here we prove that $\partial_3\partial_\beta(\alpha) = \partial_\beta\partial_3(\alpha)$ if

$$\begin{array}{lll}
\beta \in (a, a, a) & \text{and} & \alpha \in \mathcal{D}_c \\
\beta \in (a, a, b) & \text{and} & \alpha \in \mathcal{D}_c \backslash (a, a, c) \\
\beta \in (a, b, b) & \text{and} & \alpha \in \mathcal{D}_c \backslash (a, a, c) \\
\beta \in (b, b, b) & \text{and} & \alpha \in \mathcal{D}_c \\
\beta \in (a, a, c) & \text{and} & \alpha \in \mathcal{D}_c \\
\beta \in (a, b, c) & \text{and} & \alpha \in \mathcal{D}_c \\
\beta \in (b, b, c) & \text{and} & \alpha \in \mathcal{D}_c \\
\beta \in (c, c, c) & \text{and} & \alpha \in \mathcal{D}_c
\end{array}.$$

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Received 01.09.2017 **Reviewer:** Assoc. prof. PhD Mariana Durcheva

Годишник на Технически Университет - София, том 67, книга 3, 2017
Proceedings of the Technical University of Sofia, Volume 67, Issue 3, 2017

КОЛИЧЕСТВЕНИ ОЦЕНКИ НА АЛГОРИТМИЧНИТЕ ВЪЗМОЖНОСТИ НА ПРОМИШЛЕНИТЕ РЕГУЛАТОРИ - част 1

Емил Николов

Анотация: *Темата на тази работа е създаването на метод и алгоритъм за количествено оценка на качеството чрез функционалните възможности на индустриалните контролери. Изложени са стандартните норми за оценка на нормалната работна област на индустриалните контролери и аналитичните методи и алгоритми за тяхното определяне. Използват се честотните характеристики на контролерите или баластните динамични системи. Предлагат се решенията с техните* ***2D-***, ***3D-****визуализации на конкретни цифрови примери за окончателните функционални оценки на индустриалните контролери.*
Ключови думи: *технически средства за автоматизация, метод за количествена оценка на функционалните възможности на индустриалните контролери, зона на нормална експлоатация.*

QUANTITATIVE ESTIMATION OF ALGORITHMIC POSSIBILITIES OF INDUSTRIAL CONTROLLERS - part 1

Emil Nikolov

Abstract: *The theme of this working is a creating a method and algorithm for quantitative estimation of the quality by functional possibilities of the industrial controllers. The standard norms for the estimation of the normal operation domain of the industrial controllers and the analytical methods and algorithms for their determination are described. It is used the frequency characteristics of the controllers or a ballast dynamic systems. The solutions with their* ***2D-***, ***3D-****visualizations of concrete numeric examples for the final functional estimations of the industrial controllers are proposed.*
Key words: *control instrumentation, method for quantitative estimation of the functional possibilities of the industrial controllers, area of normal operation.*

ВЪВЕДЕНИЕ

Между реалните функционални възможности на серийно произвежданите (промишлени) технически средства за автоматизация (***ТСА***) и еталонните алгоритми за управление и регулиране съществуват различия, основаващи се преди всичко на реализацията на промишлените ***ТСА*** с електрически, цифрови, пневматични, хидравлични и механични градивни елементи.

Факторите **[1 ÷ 34]**, определящи тези различия, могат да се обобщят в две групи.

ISSN 1311-0829

Вътрешните фактори са предопределени с наличието на: ● механично триене и натоварване; механични вибрации; ● механични луфтове; механични ограничения - геометрични в пространството; ● механични ограничения - ограничения в движението (движение с постоянна скорост; движение с ограничена от натоварването скорост и др.); ● маса и подвижни елементи; ● еластични елементи; ● стареене на материалите; ● вискозно триене на използвания работен несвиваем флуид; ● изменящи се режими на движение на работния флуид (ламинарно, турбулентно движение, пулверизация, кавитация, суперкавитация, хидравличен удар при несвиваем работен флуид); ● изменящи се режими на движение на работния флуид (звук от до- и над- звукова скорост на движение на свиваем работен флуид); ● изменящи се режими на движение на работния флуид (втечняване на свиваемия работен флуид и преминаване във двуфазна среда); ● хидравлично натоварване; ● дискретизация на сигналите по ниво и/или по време; ● схемната структура и реализиращата технология (електрическа, пневматична, хидравлична, комбинирана; непрекъсната, дискретна, релейноимпулсна, цифрова) на реализация. **Външните фактори** се обобщават с наличието на: ● флуктуации в енергозахранването (електрическо, пневматично, хидравлично); ● флуктуации на температурата, влажността, налягането (атмосфера, хидросфера) на околната среда; ● интензивността на лъчение; ● полета (магнитни и електромагнитни) и радиационен фон на околната среда; ● механичната абразивност; химическа и биологическа и агресивност на околната среда и на работната среда (управление в условията на промишленото производство на хидродинамични параметри на суспензии, пулпове, основи, киселини и др.).

Различията на свойствата и на алгоритмичните възможности на промишлените регулатори от съответстващите им еталонните алгоритми, се оценяват количествено с помощта на: областта на нормална работа (***ОНР***) за аналоговите регулатори; областта на линейни режими (***ОЛР***) за импулсните и релейно-импулсните регулатори; областта на функциониране без пулсации (***ОФБП***) за цифровите регулатори. Известни са [1 ÷ 34] дефинициите на тези оценки по международните стандарти. Приема се, че за онази част от параметричните множества на параметрите за настройка, честотата, входния сигнал и такта на дискретизация по време, за които не са изпълнени изискванията на ***ОНР*** (или съответно на ***ОЛР***, или на ***ОФБП***) от оценявания промишлен регулатор, той не притежава изискващите се от него алгоритмични възможности. Количествените оценки за алгоритмичните възможности се използват за прецизен анализ на конкретни промишлени (реални) регулатори или за прецизен сравнителен анализ (или избор) на промишлени регулатори от един и същи клас.

Разработката е представена в две неразривно свързани части. Настоящата е първата от тях и включва: въведение, цел и постановка на задачата, основни дефиниции, аналитична постановка на задачите в изследването, числени примери, подход, метод и алгоритъм за функционална параметризация на свойствата на баластните звена на реалните регулатори за определяне на тяхната ***ОНР***. Втората част включва резултати от изследването, дискусия и анализ на резултатите, заключение и изводи, литература.

ЦЕЛ И ПОСТАНОВКА НА ЗАДАЧАТА

Настоящата работа цели да предложи подход, аналитичен метод и реализиращ го алгоритъм за определяне на количествени оценки на алгоритмичните възможности на промишлените регулатори, като си поставя задачата при известни структури на промишлените регулатори да анализира техните алгоритмични възможности и определи ***ОНР***.

ОСНОВНИ ДЕФИНИЦИИ

Нека свойствата на оценявания ***промишлен регулатор*** R_{Reel} са означени с $R_{Reel}(j\omega, k_p, T_i, T_d, ...)$, а на съответстващия му ***еталонен регулатор*** (***алгоритъм***) R_{Etalon} - с $R_{Etalon}(j\omega, k_p, T_i, T_d, ...)$. Тогава ***ОНР*** на R_{Reel} се определя аналитично като функция на параметричното множество $\mathcal{R} = \{\omega, k_P, T_i, T_d\}$ за всички стойности на честотата ω и на динамичните параметри ($k_P, T_i, T_d, ...$), за които е изпълнено изискването (1). Параметричното множеството $\mathcal{R} = \{\omega, k_P, T_i, T_d\}$ се разглежда като съставно от две непресичащи се параметрични подмножества $\mathcal{R}^{\blacksquare}$ и $\mathcal{R}^{\vee}$. Първото от тях е $\mathcal{R}^{\blacksquare} = \{\omega^{\blacksquare}, k_p^{\blacksquare}, T_i^{\blacksquare}, T_d^{\blacksquare}\}$, за което изискванията на (1) не са изпълнени, в което оценяваният регулатор R_{Reel} функционира с алгоритъм, отличаващ се недопустимо от еталонния R_{Etalon} по предявените норми (1). Второто $\mathcal{R}^{\vee} = \{\omega^{\vee}, k_p^{\vee}, T_i^{\vee}, T_d^{\vee}\}$ множество удовлетворява изискванията (1) и дефинира областта на нормална работа ***ОНР*** , в която регулаторът R_{Reel} функционира с алгоритъм, отличаващ се от съответния му R_{Etalon} с не повече от допустимите норми (1).

$$\left\{ \begin{array}{c} \left| \left(\left| R_{Reel}(j\omega) \right| - \left| R_{Etalon}(j\omega) \right| \right) \left| R_{Etalon}(j\omega) \right|^{-1} \right| 100,\% \le 10,\% \\ \left| arg\left(R_{Reel}(j\omega)\right) - arg\left(R_{Etalon}(j\omega)\right) \right| \le 10^{o} \end{array} \right\} \qquad (1)$$

$$R_{Reel}(j\omega) = R_{Etalon}(j\omega)\, B(j\omega) \qquad (2)$$

$$\left\{ \begin{array}{c} 0.9 \le \left| B(j\omega) \right| \le 1.1 \\ \left| arg\left(B(j\omega)\right) \right| \le 10^{o} \end{array} \right\} \qquad (3)$$

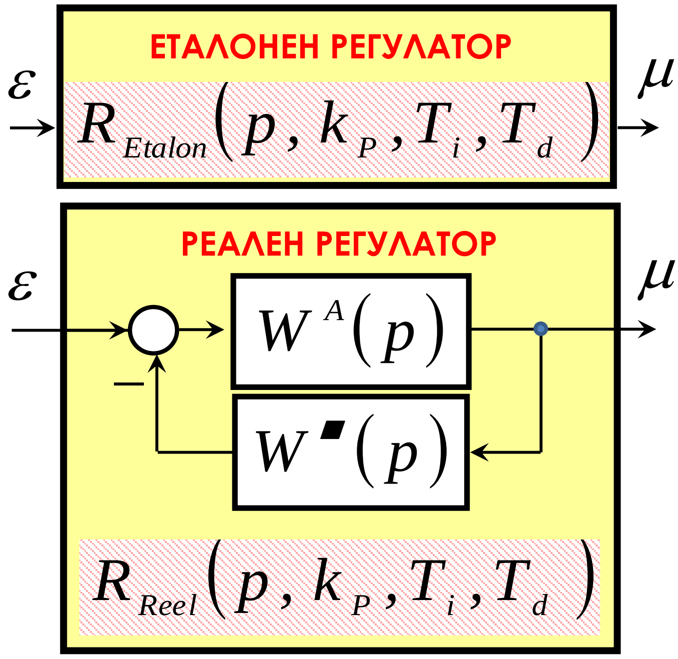
Възможно е характеристиката на оценявания промишлен регулатор R_{Reel} да се представи като фиктивна динамична система (2) от последователно съединение на две звена - еталонен закон R_{Etalon} (съответстващ на R_{Reel}) и динамично допълнение, наречено ***баластно звено*** B, с което реализацията на промишления регулатор R_{Reel} се отличава от R_{Etalon}. Тогава изискванията (1) към R_{Reel}, отнесени към R_{Etalon}, се трансформират еквивалентно в изискването (3) към B. По изложената по-горе логика, $\mathcal{B} = \{\omega, k_P, T_i, T_d\}$ е парамеричното множество, характеризиращо баластното звено B. В контекста на дефиницията (1), могат да бъдат формирани две непресичащи се множества в $\mathcal{B}$. Първото от тях е $\mathcal{B}^{\blacksquare}$ (4), което не удовлетворява (3) и второто - $\mathcal{B}^{\vee}$ (5), удовлетворяващо изискванията (1) и което дефинира областта на нормална работа ***ОНР*** на регулатора R_{Reel}.

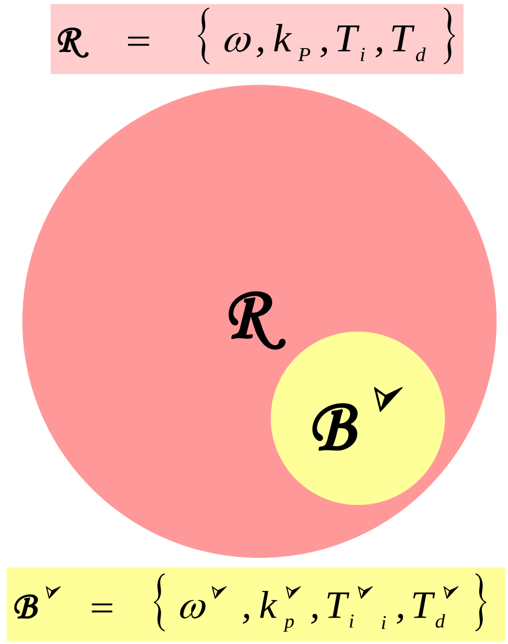
$$\mathcal{B}^{\blacksquare} = \{\omega^{\blacksquare}, k_p^{\blacksquare}, T_i^{\blacksquare}, T_d^{\blacksquare}\}; \quad \mathcal{B}^{\blacksquare} \subseteq \mathcal{R}^{\blacksquare} \qquad (4)$$

$$\mathcal{B}^{\vee} = \{\omega^{\vee}, k_p^{\vee}, T_i^{\vee}, T_d^{\vee}\}; \quad \mathcal{B}^{\vee} \subseteq \mathcal{R}^{\vee} \qquad (5)$$

АНАЛИТИЧНА ПОСТАНОВКА НА ЗАДАЧИТЕ В ИЗСЛЕДВАНЕТО

Илюстрацията на структурната постановката за определяне на ***ОНР*** (фиг.1) отразява сравняваните реален R_{Reel} (6) и съответстващия му еталонен R_{Etalon} регулатори (7) с отчитане на (8) ÷ (13) .





Фиг.1. **Фиг.2.**

$$\left\{ \begin{array}{c} R_{Reel}(j\omega) = W^{A}\left(1 + W^{A} W^{\blacksquare}\right)^{-1} = Mod_{Reel}(\omega)\, e^{-j\, Arg_{Reel}(\omega)}, \\ \left(\begin{array}{c} Mod_{Reel}\left(\omega, k_P, T_i, T_d\right) = \left| R_{Reel}(j\omega) \right| \\ Arg_{Reel}\left(\omega, k_P, T_i, T_d\right) = Arg\left(R_{Reel}(j\omega)\right) \end{array} \right) \end{array} \right\} \qquad (1)$$

$$\left\{ \begin{array}{c} R_{Etalon}(j\omega) = Mod_{Etalon}(\omega)\, e^{-j\, Arg_{Etalon}(\omega)}, \\ \left(\begin{array}{c} Mod_{Etalon}\left(\omega, k_P, T_i, T_d\right) = \left| R_{Etalon}(j\omega) \right| \\ Arg_{Etalon}\left(\omega, k_P, T_i, T_d\right) = Arg\left(R_{Etalon}(j\omega)\right) \end{array} \right) \end{array} \right\} \qquad (2)$$

$$B\left(j\omega, k_P, T_i, T_d\right) = Re_B\left(\omega, k_P, T_i, T_d\right) + j\, Im_B\left(\omega, k_P, T_i, T_d\right) \qquad (3)$$

$$B\left(j\omega, k_P, T_i, T_d\right) = Mod_B\left(\omega, k_P, T_i, T_d\right) \exp\left(-j\, Arg_B\left(\omega, k_P, T_i, T_d\right)\right) \qquad (4)$$

$$Mod_B\left(\omega, k_p, T_d\right) = \left(Re_B^2\left(\omega, k_p, T_d\right) + Im_B^2\left(\omega, k_p, T_d\right) \right)^{0.5} \qquad (5)$$

$$Arg_B\left(\omega, k_P, T_i, T_d\right) = arctg\left(-Im_B\left(\omega, k_P, T_i, T_d\right) / Re_B\left(\omega, k_P, T_i, T_d\right) \right) \qquad (6)$$

$$Re_B\left(\omega, k_P, T_i, T_d\right) = Mod_B\left(\omega, k_P, T_i, T_d\right)\, Cos\left(Arg_B\left(\omega, k_P, T_i, T_d\right)\right) \qquad (7)$$

$$Im_B\left(\omega, k_P, T_i, T_d\right) = Mod_B\left(\omega, k_P, T_i, T_d\right)\, Sin\left(Arg_B\left(\omega, k_P, T_i, T_d\right)\right) \qquad (8)$$

където R_{Reel} е конфигуриран по структура (фиг.1) с усилвател W^{A} и обратна връзка $W^{\blacksquare}$, а (14) е параметричното множество $\mathcal{R}$, характеризиращо промишлените (реалните) регулатори.

$$\mathcal{R} = \left\{ \omega, k_P, T_i, T_d \right\}, \quad \left(\begin{array}{c} k_p \in \left[k_{p,min}, k_{p,max} \right];\ T_i \in \left[T_{i,min}, T_{i,max} \right]; \\ T_d \in \left[T_{d,min}, T_{d,max} \right];\ \omega \in [0, \infty) \end{array} \right) \qquad (14)$$

Баластното звено B (15), аналитично определя с размера на характеристиките си алгоритмичната близост на R_{Reel} до R_{Etalon}.

$$\left\{\begin{array}{c} B\left(j\omega,k_p,T_i,T_d\right)=\dfrac{R_{Etalon}\left(j\omega,k_p,T_i,T_d\right)}{R_{Reel}\left(j\omega,k_p,T_i,T_d\right)} \\ B=R_{Etalon}\,R^{\Diamond}_{Reel}\quad,\ \left(R^{\Diamond}_{Reel}=\left(R_{Reel}\right)^{-1}\right) \\ B\left(j\omega,k_p,T_i,T_d\right)=Mod_B\left(\omega,k_p,T_i,T_d\right)e^{-j\,Arg_B\left(\omega,k_p,T_i,T_d\right)} \\ \left(Mod_B\left(\omega,k_p,T_i,T_d\right)=\left|B\left(j\omega\right)\right|;\ Arg_B\left(\omega,k_p,T_i,T_d\right)=Arg\left(B\left(j\omega\right)\right)\right) \end{array}\right\} \tag{15}$$

Въз основа на изложеното (6)÷(15), параметрично множество $\mathcal{B}^{\vee}$, като еднозначна аналитична дефиниция на областта на нормална работа ***ОНР,*** се определя с (16)

$$\begin{array}{c} \mathcal{B}^{\vee}=\left\{\omega^{\vee},k_p^{\vee},T_i^{\vee},T_d^{\vee}:\left(Mod\,B\le 0{,}1\ \wedge\ \left|Arg\,B\right|\le 10^0\right)\right\} \\ \forall\ k_p^{\vee}\ \forall\ T_i^{\vee}\ \forall\ T_d^{\vee}\ \forall\ \omega^{\vee}\ \exists\ \mathcal{B}^{\vee} \\ \left\{\omega^{\vee},k_p^{\vee},T_i^{\vee},T_d^{\vee}\right\}\subseteq\left\{\omega,k_p,T_i,T_d\right\}\ ;\ \mathcal{B}^{\vee}\subseteq\ \mathcal{R} \end{array} \tag{16}$$

Съотношението между параметричните множества (14), (16) е илюстрирано на фиг.2, тъй като е в сила (17), т.е. че множеството (16) принадлежи на (14).

$$\left\{\omega^{\vee},k_p^{\vee},T_i^{\vee},T_d^{\vee}\right\}\subseteq\left\{\omega,k_p,T_i,T_d\right\},\ \left(\mathcal{B}^{\vee}\subseteq\ \mathcal{R}\right) \tag{17}$$

За определяне на ***ОНР*** настоящата работа използва ***метода функционалната параметризация***. Неговата идея **[35-61]**, като традиционна приложна задача в инженерната практика (при априори известни характеристики (18) на динамични системи като функции на повече от една променливи $\mathcal{E}\left(\varsigma_1,\varsigma_2,\varsigma_3,\ldots\right)$) е търсенето, аналитичното определяне и анализът на функционалната зависимост (19) на конкретни показатели $\mathcal{F}\left(\mathcal{E}\right)$ (***ОНР*** като показател на качеството) на динамичните системи от променливите $\left(\varsigma_1,\varsigma_2,\varsigma_3,\ldots\right)$ или и от техните вариации и следваща трансформация на (19) до (20) с цел достигането до (21). Във функционалния анализ тя е известна като „*задача за функционална параметризация*“ на показатели (фактори, свойства) на функции и е изразена с $\mathcal{F}=\mathcal{F}\left(\varsigma_1,\varsigma_2,\varsigma_3,\ldots\right)$ (20), от която след обратна трансформация се достига до крайната зависимост (21). Последната е точното решение на поставената задача за определяне на ***ОНР*** - $\mathcal{B}^{\vee}$ (16)

$$\mathcal{E}\left\{\varsigma_1,\varsigma_2,\varsigma_3,\ldots\right\}\ \Leftrightarrow\ \mathcal{R}\left\{\omega_1,k_p,T_i,T_d\right\} \tag{18}$$

$$\mathcal{F}\left(\mathcal{E}\right)\quad\Leftrightarrow\quad\mathcal{B}^{\vee}\subseteq\ \mathcal{R} \tag{19}$$

$$\mathcal{F}=\mathcal{F}\left\{\varsigma_1,\varsigma_2,\varsigma_3,\ldots\right\}\Leftrightarrow\mathcal{B}^{\vee}=\left\{\omega^{\vee},k_p^{\vee},T_i^{\vee},T_d^{\vee}\right\} \tag{20}$$

$$\left\{\begin{array}{c} \varsigma_1=\varsigma_1\left(\varsigma_2,\varsigma_3\right),\left(\mathcal{F}=const\in\left[\mathcal{F}_{min},\mathcal{F}_{max}\right]\right)\Leftrightarrow \\ \Leftrightarrow\ \omega^{\vee}=\omega^{\vee}\left(k_p^{\vee},T_i^{\vee},T_d^{\vee}\right),\left(\mathcal{B}^{\vee}=const\in\left[\mathcal{B}^{\vee\,0,9\le Mod}_{\ 10^0\ge Arg},\mathcal{B}^{\vee\,Mod\ \le 1,1}_{\ 10^0\ge Arg}\right]\right) \\ \\ \varsigma_2=\varsigma_2\left(\varsigma_1,\varsigma_3\right),\left(\mathcal{F}=const\in\left[\mathcal{F}_{min},\mathcal{F}_{max}\right]\right)\Leftrightarrow \\ \Leftrightarrow\ k_p^{\vee}=k_p^{\vee}\left(\omega^{\vee},T_i^{\vee},T_d^{\vee}\right),\left(\mathcal{B}^{\vee}=const\in\left[\mathcal{B}^{\vee\,0,9\le Mod}_{\ 10^0\ge Arg},\mathcal{B}^{\vee\,Mod\ \le 1,1}_{\ 10^0\ge Arg}\right]\right) \\ \\ \varsigma_3=\varsigma_3\left(\varsigma_1,\varsigma_2\right),\left(\mathcal{F}=const\in\left[\mathcal{F}_{min},\mathcal{F}_{max}\right]\right)\Leftrightarrow \\ \Leftrightarrow\ T_i^{\vee}=T_i^{\vee}\left(\omega^{\vee},k_p^{\vee},T_d^{\vee}\right),\left(\mathcal{B}^{\vee}\in\left[\mathcal{B}^{\vee\,0,9\le Mod}_{\ 10^0\ge Arg},\mathcal{B}^{\vee\,Mod\ \le 1,1}_{\ 10^0\ge Arg}\right]\right) \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \end{array}\right\} \tag{21}$$

ЧИСЛЕНИ ПРИМЕРИ

Въз основа на възможностите за структурна конфигурация (фиг.1), в работа като числени примери са решени в символен вид дванадесет представителни задачи за определяне на ***ОНР*** на реални ***PI-***, ***PD-*** и ***PID***-регулатори (систематизирани в табл.1). Избрани са аналитични модели на четири класа усилватели (22)$\div$(27) (отличаващи се съществено по характеристиките си) и три класа динамични звена (26)$\div$(28) в обратната връзка (фиг.1).

За всеки един от конфигурираните ***PI-***, ***PD-*** и ***PID***-регулатори (фиг.1; табл.1) са определени (29.a,b)$\div$(40.a,b) в символен вид характеристиките на $R_{i,algorithm}^{amplifier}$ и на принадлежащите им баластни звена $B_{i,algorithm}^{amplifier}$, отличаващи ги от съответстващия им еталонен алгоритъм. Използвани са индекси, указващи вида на усилвателя (*amplifier*) и вида на алгоритъма (*algorithm*).

Таблица 1.

усилвател W^{A}			
$W^{PP}=a$ (22)	$W^{IP}=\dfrac{a}{(j\omega)}$ (23)	$W^{AA}=\dfrac{a}{(1+b\,j\omega)}$ (24)	$W^{IA}=\dfrac{a}{b\,j\omega(1+b\,j\omega)}$ (25)
обратна връзка $W^{\blacksquare}$			
$W_{PD}^{\blacksquare}=\dfrac{k_p^{-1}}{(1+T_d\,j\omega)}$	$W_{PD}^{\blacksquare}=\dfrac{k_p^{-1}}{(1+T_d\,j\omega)}$	$W_{PD}^{\blacksquare}=\dfrac{k_p^{-1}}{(1+T_d\,j\omega)}$	$W_{PD}^{\blacksquare}=\dfrac{k_p^{-1}}{(1+T_d\,j\omega)}$ (26)
$W_{PI}^{\blacksquare}=\dfrac{k_p^{-1}T_i\,j\omega}{(1+T_i\,j\omega)}$	$W_{PI}^{\blacksquare}=\dfrac{k_p^{-1}T_i\,j\omega}{(1+T_i\,j\omega)}$	$W_{PI}^{\blacksquare}=\dfrac{k_p^{-1}T_i\,j\omega}{(1+T_i\,j\omega)}$	$W_{PI}^{\blacksquare}=\dfrac{k_p^{-1}T_i\,j\omega}{(1+T_i\,j\omega)}$ (27)
$W_{PID}^{\blacksquare}=\dfrac{k_p^{-1}\,T_i\,j\omega}{(1+T_i\,j\omega)(1+T_d\,j\omega)}$	$W_{PID}^{\blacksquare}=\dfrac{k_p^{-1}\,T_i\,j\omega}{(1+T_i\,j\omega)(1+T_d\,j\omega)}$	$W_{PID}^{\blacksquare}=\dfrac{k_p^{-1}\,T_i\,j\omega}{(1+T_i\,j\omega)(1+T_d\,j\omega)}$	$W_{PID}^{\blacksquare}=\dfrac{k_p^{-1}\,T_i\,j\omega}{(1+T_i\,j\omega)(1+T_d\,j\omega)}$ (28)

Инверсните характеристики $R_{i,algorithm}^{\lozenge\,amplifier}$ на реалните ***PI-***, ***PD-*** и ***PID***-регулатори, основаващи са на W^{AA} (24) и на W^{IA} (25), съответстващи им (фиг.1; табл.1) еталонните регулатори $R_{i,Etalon}$ и баластни звена $B_{i,algorithm}^{amplifier}$ (31)-(32), (35)-(36), (39)-(40) са моделирани. Резултатите от симулацията за диапазони на вариация (36) на параметричното множество $\mathcal{R}$ (14) на моделите са визуализирани на фиг.3$\div$фиг.8.

За всяка от характеристиките на илюстрираните структури (фиг.3$\div$фиг.8, фиг.1; табл.1) е очевидно наличието на:

- ***параметрични области*** $\mathcal{B}^{\blacksquare}$ със значими недопустими по норма различия в модулната и аргументната съставящи на баластните звена $B_{i,algorithm}^{amplifier}$, чиито свойства се описват с (42) и не удовлетворяват изискванията (16);

$$R_{PD}^{PP} = \frac{a\ k_p\left(1+T_d\, j\omega\right)}{a + k_p + k_p T_d\, j\omega} \qquad (29.a)$$

$$B_{PD}^{PP} = \frac{a + k_p T_d\, j\omega}{a} \qquad (29.b)$$

$$B_{PD}^{IP} = \frac{a\ k_p\left(1+T_d\, j\omega\right)}{a + k_p\, j\omega\left(1+T_d\, j\omega\right)} \qquad (30.a)$$

$$B_{PD}^{IP} = \frac{a + k_p\, j\omega\left(1+T_d\, j\omega\right)}{a} \qquad (30.b)$$

$$R_{PD}^{AA} = \frac{a\ k_p\left(1+T_d\, j\omega\right)}{a+a\ k_p\left(1+b\, j\omega\right)\left(1+T_d\, j\omega\right)} \qquad (31.a)$$

$$B_{PD}^{AA} = \frac{a + k_p\left(1+b\, j\omega\right)\left(1+T_d\, j\omega\right)}{a} \qquad (31.b)$$

$$R_{PD}^{AI} = \frac{a\ k_p\left(1+T_d\, j\omega\right)}{a+a\ k_p\, j\omega\left(1+b\, j\omega\right)\left(1+T_d\, j\omega\right)} \qquad (32.a)$$

$$B_{PD}^{AI} = \frac{a + k_p\, j\omega\left(1+b\, j\omega\right)\left(1+T_d\, j\omega\right)}{a} \qquad (32.b)$$

$$R_{PI}^{PP} = \frac{a\ k_p\, T_i\, j\omega}{a + a\, T_i\, j\omega + k_p\, T_i\, j\omega} \qquad (33.a)$$

$$B_{PI}^{PP} = \frac{\left(a + a\, T_i\, j\omega + k_p\, T_i\, j\omega\right)}{a\left(T_i\, j\omega\right)^2\left(1+T_d\, j\omega\right)^{-1}} \qquad (33.b)$$

$$R_{PI}^{IP} = \frac{a\, k_p\, T_i\left(j\omega\right)}{a+a\, T_i\, j\omega+k_p T_i\left(j\omega\right)^2} \qquad (34.a)$$

$$B_{PI}^{IP} = \frac{\left(a + a\, T_i\, j\omega + k_p\, T_i\left(j\omega\right)^2\right)}{a\left(T_i\, j\omega\right)^2\left(1+T_d\, j\omega\right)^{-1}} \qquad (34.b)$$

$$R_{PI}^{AA} = \frac{a\ k_p\, T_i\, j\omega}{a+a\, T_i\, j\omega+k_p T_i\, j\omega\left(1+b\, j\omega\right)} \qquad (35.a)$$

$$B_{PI}^{AA} = \frac{\left(a + a\, T_i\, j\omega + k_p T_i\, j\omega\left(1+b\, j\omega\right)\right)}{a\left(T_i\, j\omega\right)^2\left(1+T_i\, j\omega\right)^{-1}} \qquad (35.b)$$

$$R_{PI}^{IA} = \frac{a\ k_p\, T_i\, j\omega}{a+a\ T_i\, j\omega+b\ k_p T_i\left(j\omega\right)^2\left(1+b\, j\omega\right)} \qquad (36.a)$$

$$B_{PI}^{IA} = \frac{\left(a + a\, T_i\, j\omega + b\ k_p\, T_i\left(j\omega\right)^2\left(1+b\, j\omega\right)\right)}{a\left(T_i\, j\omega\right)^2\left(1+T_i\, j\omega\right)^{-1}} \qquad (36.b)$$

$$R_{PID}^{PP} = \frac{a\left(k_p + k_p T_i\, j\omega + k_p T_d\, j\omega + k_p T_i T_d\left(j\omega\right)^2\right)}{a\ T_i\, j\omega + k_p + k_p T_i\, j\omega + k_p T_d\, j\omega + k_p T_i T_d\left(j\omega\right)^2} \qquad (37.a)$$

$$B_{PID}^{PP} = \frac{a\, T_i\, j\omega + k_p\left(1+T_i\, j\omega\right)\left(1+T_d\, j\omega\right)}{a\ T_i\, j\omega} \qquad (37.b)$$

$$R_{PID}^{IP} = \frac{a\left(k_p + k_p T_i\, j\omega + k_p T_d\, j\omega + k_p T_i T_d\left(j\omega\right)^2\right)\left(j\omega\right)^{-1}}{\left(a\ T_i + \left(k_p + k_p T_i\, j\omega + k_p T_d\, j\omega + k_p T_i T_d\left(j\omega\right)^2\right)\right)} \qquad (38.a)$$

$$B_{PID}^{IP} = \frac{\left(a\, T_i + k_p\left(1+T_i\, j\omega\right)\left(1+T_d\, j\omega\right)\right)}{a\ T_i} \qquad (38.b)$$

$$R_{PID}^{AA} = \frac{a\ k_p\left(1+T_i\, j\omega\right)\left(1+T_d\, j\omega\right)}{a\ T_i + k_p\left(1+b\ j\omega\right)\left(1+T_i\, j\omega\right)\left(1+T_d\, j\omega\right)} \qquad (39.a)$$

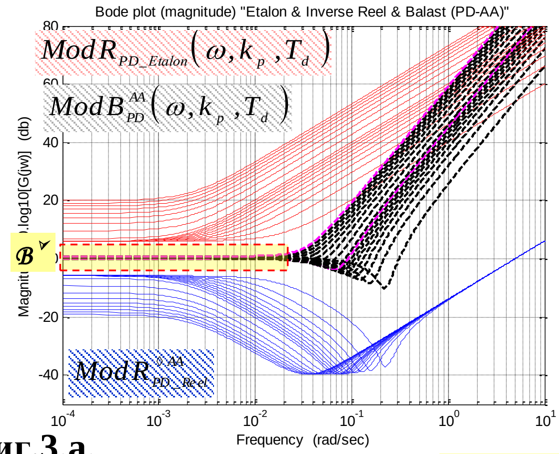
$$B_{PID}^{AA} = \frac{a\ T_i\, j\omega + k_p\left(1+b\, j\omega\right)\left(1+T_i\, j\omega\right)\left(1+T_d\, j\omega\right)}{a\ T_i\, j\omega} \qquad (39.b)$$

$$R_{PID}^{IA} = \frac{a\ k_p\left(1+T_i\, j\omega\right)\left(1+T_d\, j\omega\right)}{j\omega\left(A + B\right)},$$
$$\left(A = a\ T_i + b\ k_p\left(1+T_i\, j\omega\right)\left(1+T_d\, j\omega\right)\right),$$
$$\left(B = b^2\, k_p\, j\omega\left(1+T_i\, j\omega\right)\left(1+T_d\, j\omega\right)\right) \qquad (40.a)$$

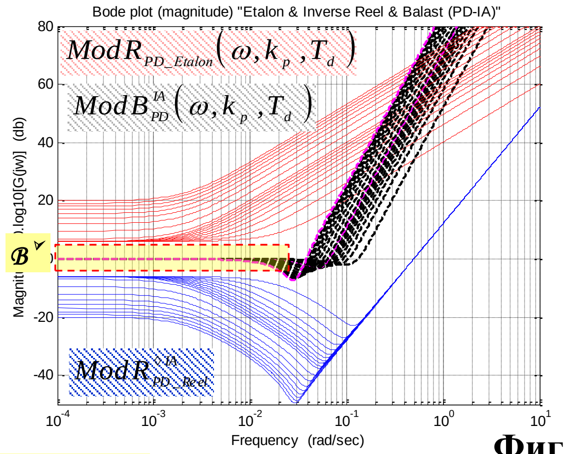
$$B_{PID}^{IA} = \frac{j\omega\left(a\, T_i + b\, k_p\left(1+b\, j\omega\right)\left(1+T_i\, j\omega\right)\right)}{a\ T_i\, j\omega\left(1+T_d\, j\omega\right)^{-1}} + \frac{j\omega\left(b^2\, k_p\ j\omega\left(1+T_i\, j\omega\right)\left(1+T_d\, j\omega\right)\right)}{a\ T_i\, j\omega} \qquad (40.b)$$

$$0{,}05 \le k_p \le 100{,}00\, ;\ 0{,}05 \le T_i \le 500{,}00\, ;\ 0{,}05 \le T_i \le 500{,}00\, ;\ a = 200;\ b = 20 \qquad (41)$$

$$\mathcal{B}^{\blacksquare} = \left\{ \omega^{\blacksquare}, k_p^{\blacksquare}, T_i^{\blacksquare}, T_d^{\blacksquare}\ :\ \left(Mod\ B_{i,algorithm}^{amplifier} > 0{,}1\ \wedge\ \left| Arg\ B_{i,algorithm}^{amplifier} \right| > 10^0 \right) \right\}$$
$$\forall\ k_p^{\blacksquare}\ \forall\ T_i^{\blacksquare}\ \forall\ T_d^{\blacksquare}\ \forall\ \omega^{\blacksquare}\ \exists\ \mathcal{B}^{\blacksquare} \qquad (42)$$
$$\left\{ \omega^{\blacksquare}, k_p^{\blacksquare}, T_i^{\blacksquare}, T_d^{\blacksquare} \right\} \subseteq \left\{ \omega, k_p, T_i, T_d \right\}\ ;\ \mathcal{B}^{\blacksquare} \subseteq \mathcal{R}^{\blacksquare}$$

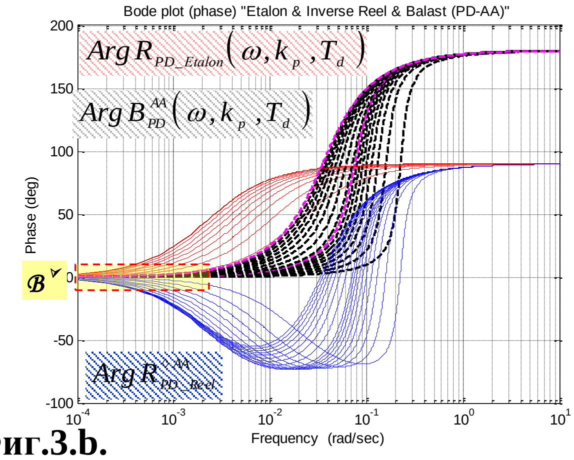


Фиг.3.a.

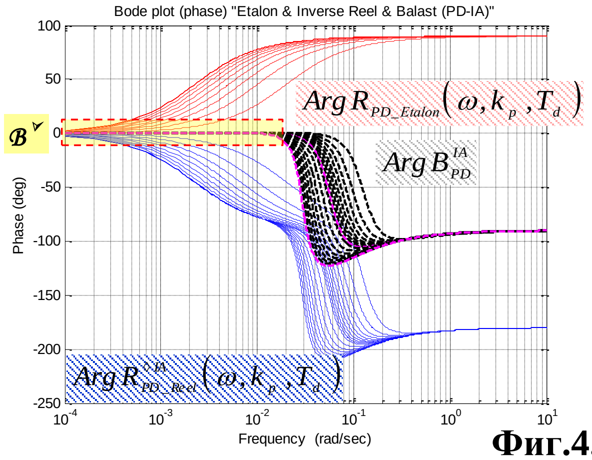


Фиг.4.a.

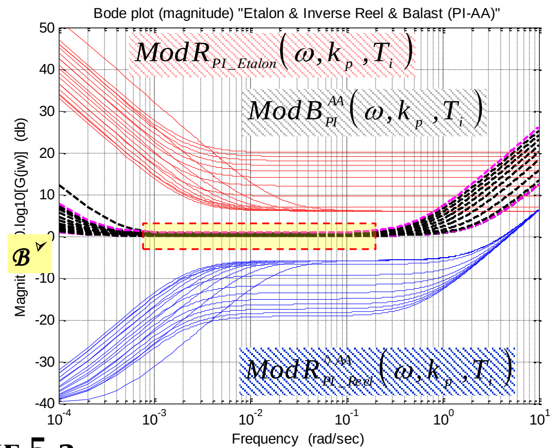
$0{,}05 \le k_p \le 100{,}00 \;\; ; \;\; 0{,}05 \le T_d \le 500{,}00$



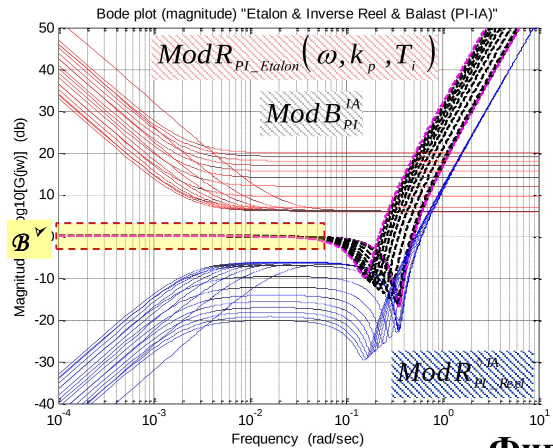
Фиг.3.b.



Фиг.4.b.

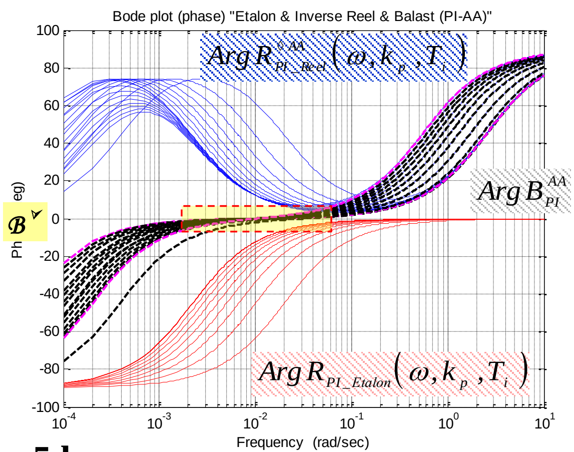


Фиг.5.a.

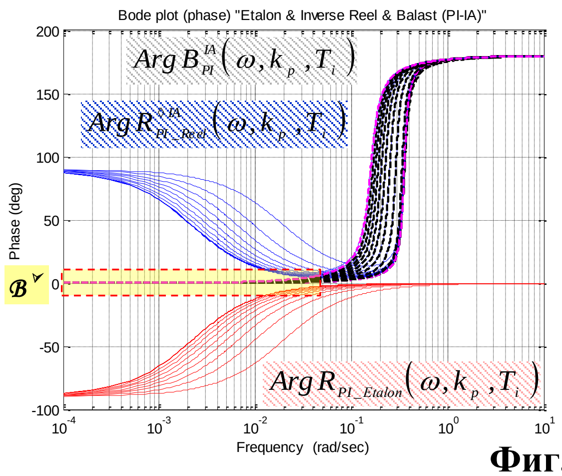


Фиг.6.a.

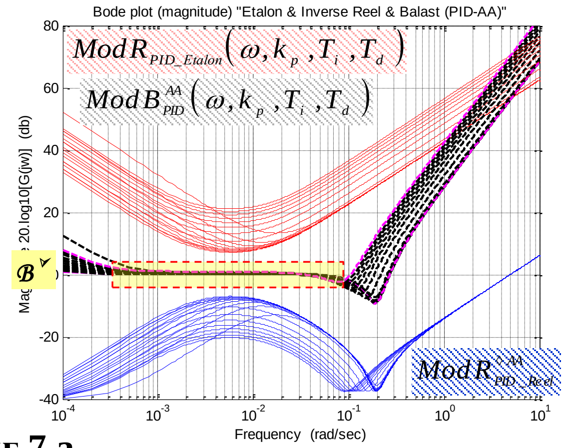
$0{,}05 \le k_p \le 100{,}00 \;\; ; \;\; 0{,}05 \le T_i \le 500{,}00$



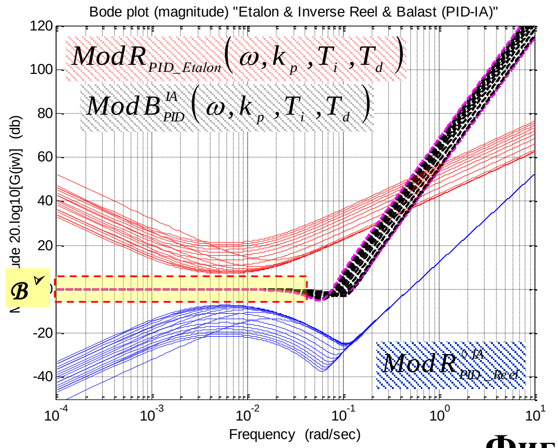
Фиг.5.b.



Фиг.6.b.

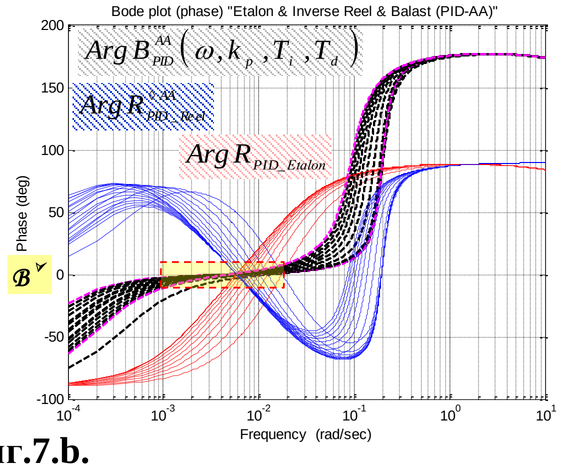


Фиг.7.a.

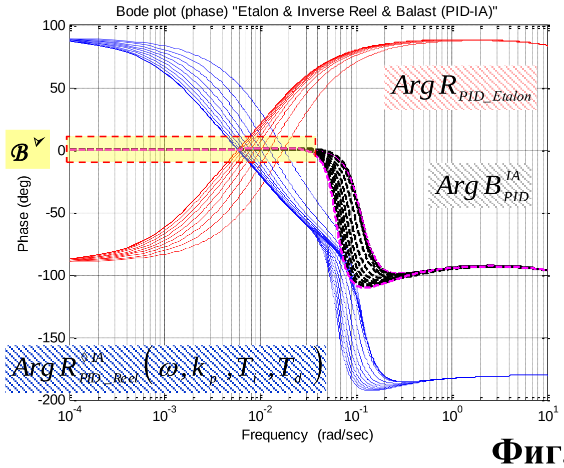


Фиг.8.a.

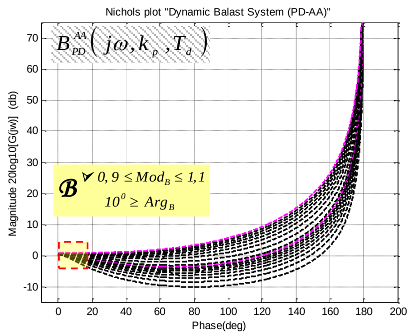
$0{,}05 \le k_p \le 100{,}00$; $0{,}05 \le T_i \le 500{,}00$; $T_d = 66{,}00 = const$



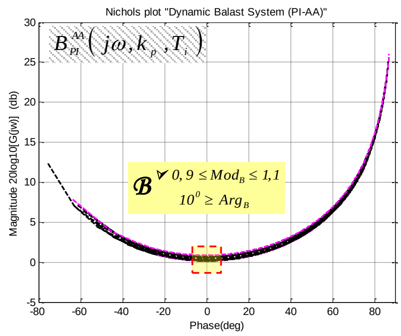
Фиг.7.b.



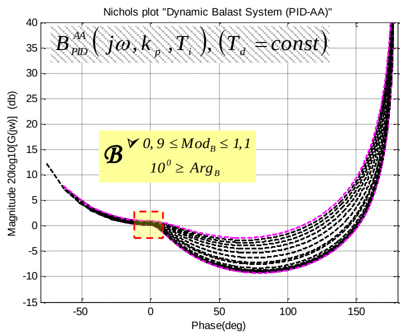
Фиг.8.b.



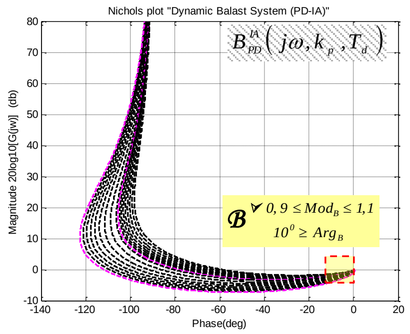
Фиг.3.c.



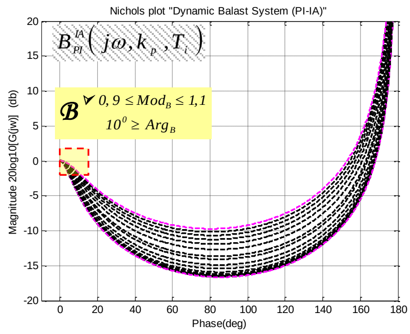
Фиг.5.c.



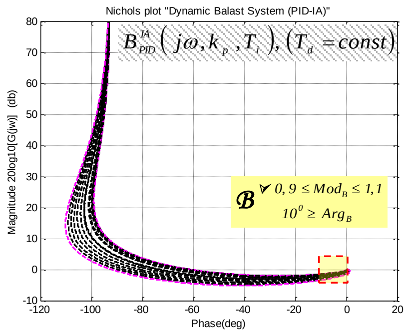
Фиг.7.c.



Фиг.4.c.



Фиг.6.c.



Фиг.8.c.

• ***параметрични области*** $\mathcal{B}^{\vee}$ (показани със защриховани зони на фиг.3 ÷ фиг.8), за които свойствата на $B_{i,algorithm}^{amplifier}$ (43) удовлетворяват напълно изискванията, формулирани с (16) за ***ОНР*** на разглежданите реални ***PI-***, ***PD-*** и ***PID***-регулатори;

$$\mathcal{B}^{\vee} = \left\{ \omega^{\vee}, k_p^{\vee}, T_i^{\vee}, T_d^{\vee} : \left(Mod\, B_{i,algorithm}^{amplifier} \le 0{,}1 \wedge \left| Arg\, B_{i,algorithm}^{amplifier} \right| \le 10^0 \right) \right\}$$
$$\forall k_p^{\vee}\ \forall T_i^{\vee}\ \forall T_d^{\vee}\ \forall \omega^{\vee}\ \exists \mathcal{B}^{\vee} \qquad (43)$$
$$\left\{ \omega^{\vee}, k_p^{\vee}, T_i^{\vee}, T_d^{\vee} \right\} \subseteq \left\{ \omega, k_p, T_i, T_d \right\} \;;\; \mathcal{B}^{\vee} \subseteq \mathcal{R}^{\vee}$$

• ***значими различия в свойствата на баластните звена*** (фиг.3.с. ÷ фиг.8.с.) на разглежданите реални ***PI-***, ***PD-*** и ***PID***-конфигурации, които показват различен размер на близост между $R_{i,algorithm}^{amplifier}$ и съответстващия му еталонен алгоритъм $R_{i,Etalon}$ за еднотипните алгоритми при критерий (43) във функция от използвания усилвател при една и съща динамика на обратната връзка $w^{\blacksquare}$ (26) ÷ (28) в структурата.

ПОДХОД, МЕТОД И АЛГОРИТЪМ ЗА ФУНКЦИОНАЛНА ПАРАМЕТРИЗАЦИЯ НА СВОЙСТВАТА НА БАЛАСТНИТЕ ЗВЕНА НА РЕАЛНИТЕ РЕГУЛАТОРИ ЗА ОПРЕДЕЛЯНЕ НА ТЯХНАТА *ОНР*

В работата се предлага ***подход***, състоящ се в използване на предварително известен аналитичен модел на характеристиката на $B_{i,algorithm}^{amplifier}$ с цел съответстваща трансформация за определяне на желания показател на качеството (в случая на ***ОНР***) (20) на изследваните реални ***PI-***, ***PD-*** и ***PID***-регулатори с прилагане на съответния аналитичен инструментариум за функционална параметризация **[35 ÷ 61]**, до достигането на принадлежащата функционална зависимост (21).
Трансформацията (20) ⇒ (21) е реализирана в работата чрез аналитичен ***метод*** за решението на обратна задача, използващ субституционна недиференцируема оптимизация, с помощта на специални математически функции **[62 ÷ 64]** и параметрична апроксимация.

Алгоритъмът за решаване на същинската параметризация на показателите на $B_{i,algorithm}^{amplifier}$ се основава на оператори от структурното програмиране за интегрална трансформация на функции, вградени в специализираните математически пакети програми (***Mathematica***® на Wolfram Inc., ***MATLAB***® на MathWorks Inc. и др.) **[62 ÷ 64]**, които предоставят и възможността за ***2D-***, ***3D***-визуализация на решението за параметризация (21).

Във втората част на разработката са представени: резултати от изследването, дискусия и анализ на резултатите, заключение и изводи, литература.

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Годишник на Технически Университет - София, том 67, книга 3, 2017
Proceedings of the Technical University of Sofia, Volume 67, Issue 3, 2017

КОЛИЧЕСТВЕНИ ОЦЕНКИ НА АЛГОРИТМИЧНИТЕ ВЪЗМОЖНОСТИ НА ПРОМИШЛЕНИТЕ РЕГУЛАТОРИ - част 2

Емил Николов

Анотация: *Темата на тази работа е създаването на метод и алгоритъм за количествено оценка на качеството чрез функционалните възможности на индустриалните контролери. Изложени са стандартните норми за оценка на нормалната работна област на индустриалните контролери и аналитичните методи и алгоритми за тяхното определяне. Използват се честотните характеристики на контролерите или баластните динамични системи. Предлагат се решенията с техните* ***2D-***, ***3D-****визуализации на конкретни цифрови примери за окончателните функционални оценки на индустриалните контролери.*
Ключови думи: *технически средства за автоматизация, метод за количествена оценка на функционалните възможности на индустриалните контролери, зона на нормална експлоатация.*

QUANTITATIVE ESTIMATION OF ALGORITHMIC POSSIBILITIES OF INDUSTRIAL CONTROLLERS - part 2

Emil Nikolov

Abstract: *The theme of this working is a creating a method and algorithm for quantitative estimation of the quality by functional possibilities of the industrial controllers. The standard norms for the estimation of the normal operation domain of the industrial controllers and the analytical methods and algorithms for their determination are described. It is used the frequency characteristics of the controllers or a ballast dynamic systems. The solutions with their* ***2D-***, ***3D-****visualizations of concrete numeric examples for the final functional estimations of the industrial controllers are proposed.*
Key words: *control instrumentation, method for quantitative estimation of the functional possibilities of the industrial controllers, area of normal operation.*

ВЪВЕДЕНИЕ

Разработката се състои от две неразривно свързани части. Първата включва: въведение, цел и постановка на задачата, основни дефиниции, аналитична постановка на задачите в изследването, числени примери, подход, метод и алгоритъм за функционална параметризация на свойствата на баластните звена на реалните регулатори за определяне на тяхната ***ОНР***. Настоящата е втората част на разработката, представяща резултати от изследването, дискусия и анализ на резултатите, заключение и изводи, литература.

ISSN 1311-0829

РЕЗУЛТАТИ ОТ ИЗСЛЕДВАНЕТО

Инверсните характеристики $R^{\Diamond\, amplifier}_{i\,,algorithm}$ на реалните ***PI-***, ***PD-*** и ***PID***-регулатори, основаващи са на W^{AA} (24) и на W^{IA} (25), съответстващи им (фиг.1; табл.1) еталонните регулатори $R_{i\,,Etalon}$ и баластни звена $B^{amplifier}_{i\,,algorithm}$ (31)-(32), (35)-(36), (39)-(40) са моделирани. Резултатите от симулацията на моделите са визуализирани на фиг.3 $\div$ фиг.8 за диапазони на вариация (41) на параметричното множество $\mathcal{R}$ (14).

Определените области на нормална работа ***ОНР*** на анализираните реални ***PI-***, ***PD-*** и ***PID-*** регулатори (конфигурирани по фиг.1; табл.1, представени в първата част на разработката) са показани, съобразно систематизацията в табл.2

Таблица 2.

ОНР			
PI- регулатори	***PD-*** регулатори	***PID-*** регулатори	с усилвател W^{A}
фиг.9	фиг.10	фиг.11	(22) $W^{PP}=a$
фиг.12	фиг.13	фиг.14	(23) $W^{IP}=a\,(\,j\omega\,)^{-1}$
фиг.15	фиг.16	фиг.17	(24) $W^{AA}=a\,(\,1+b\,j\omega\,)^{-1}$
фиг.18	фиг.19	фиг.20	(25) $W^{IA}=a\,(\,b\,j\omega\,(\,1+b\,j\omega\,))^{-1}$

ДИСКУСИЯ И АНАЛИЗ НА РЕЗУЛТАТИТЕ

Резултатите от изследването (табл.2) позволяват достигането на конкретни количествени оценки на качеството (функционалната близост на реалните до съответните еталонни алгоритми) за всеки един от конфигурираните (фиг.1; табл.1) реални регулатори (29) $\div$ (40), както и сравнителни оценки (44) $\div$ (46) по отношение на конкретните конфигурации. В този смисъл показателни са оценките на размера на ***ОНР***, показани в табл.3 и илюстрирани на фиг.21 като обобщение на резултатите от изследването, систематизирани на фиг.11 $\div$ фиг.20.

Таблица 3.

	$\check{\omega}_{min}\,,rad/s$	$\check{\omega}_{max}\,,rad/s$	$\check{k}_{p,min}$	$\check{k}_{p,max}$	$\check{T}_{i,\,min}\,,s$	$\check{T}_{i,\,max}\,,s$	$\check{T}_{d,\,min}\,,s$	$\check{T}_{d,\,max}\,,s$
$\mathcal{B}^{\check{}(AA)}_{PD}$	***0,0000***	***0,0020***	***0,10***	***10,00***	***5,00***	***500,00***	***66,00***	***66,00***
$\mathcal{B}^{\check{}(IA)}_{PD}$	***0,0000***	***0,0010***	***0,10***	***10,00***	***5,00***	***500,00***	***66,00***	***66,00***
$\mathcal{B}^{\check{}(AA)}_{PI}$	***0,0010***	***0,0800***	***0,10***	***10,00***	***5,00***	***500,00***	***66,00***	***66,00***
$\mathcal{B}^{\check{}(IA)}_{PI}$	***0,0000***	***0,0800***	***0,10***	***10,00***	***5,00***	***500,00***	***66,00***	***66,00***
$\mathcal{B}^{\check{}(AA)}_{PID}$	***0,0003***	***0,0800***	***0,10***	***10,00***	***5,00***	***500,00***	***66,00***	***66,00***
$\mathcal{B}^{\check{}(IA)}_{PID}$	***0,0000***	***0,0060***	***0,10***	***10,00***	***5,00***	***500,00***	***66,00***	***66,00***

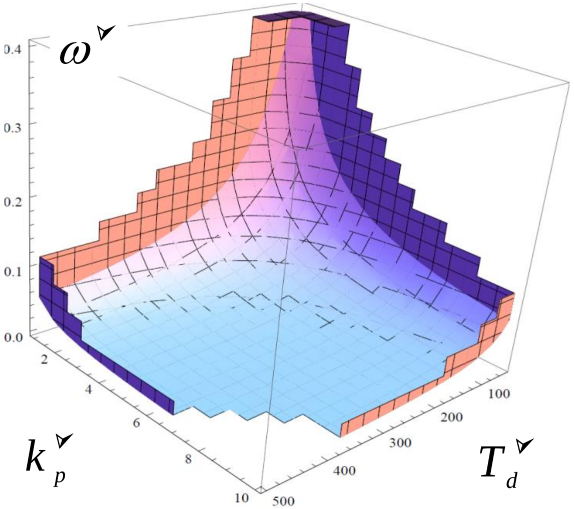
$$\mathcal{B}^{\check{}(AA)}_{PD}\left\{\,\check{\omega}\,,\check{k}_p\,,\check{T}_d\,\right\} > \mathcal{B}^{\check{}(IA)}_{PD}\left\{\,\check{\omega}\,,\check{k}_p\,,\check{T}_d\,\right\} \qquad (44)$$

$$\mathcal{B}^{\check{}(IA)}_{PI}\left\{\,\check{\omega}\,,\check{k}_p\,,\check{T}_i\,\right\} > \mathcal{B}^{\check{}(AA)}_{PI}\left\{\,\check{\omega}\,,\check{k}_p\,,\check{T}_i\,\right\} \qquad (45)$$

$$\mathcal{B}^{\check{}(IA)}_{PID}\left\{\check{\omega},\check{k}_p,\check{T}_i,\check{T}_d\,\right\} > \mathcal{B}^{\check{}(AA)}_{PID}\left\{\check{\omega},\check{k}_p,\check{T}_i,\check{T}_d\,\right\} \qquad (46)$$

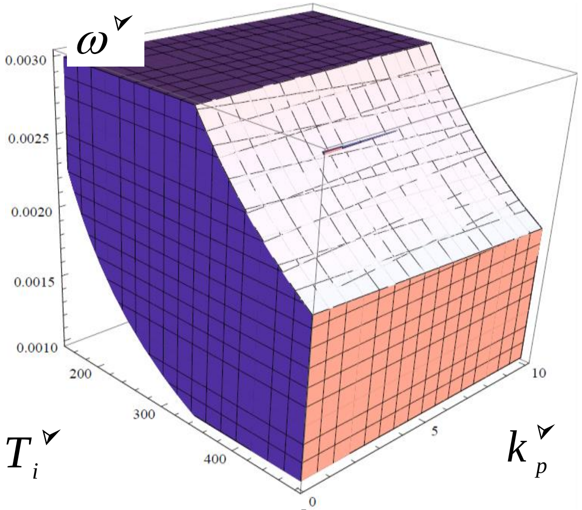
Фиг.9.

$\check{\mathcal{B}}_{PD}^{PP}\left\{\check{\omega},\check{k}_p,\check{T}_d\right\}$



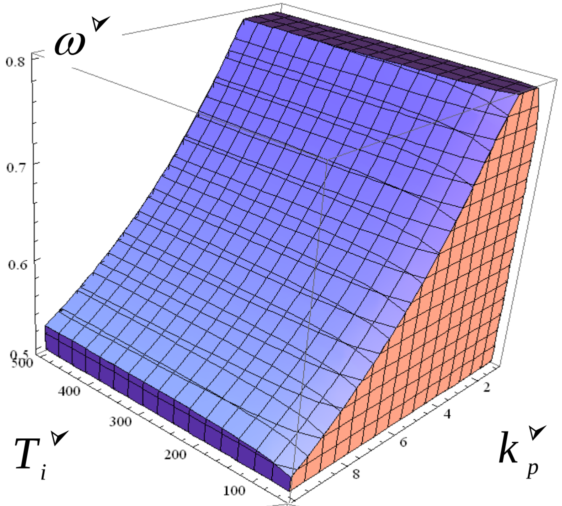
Фиг.10.

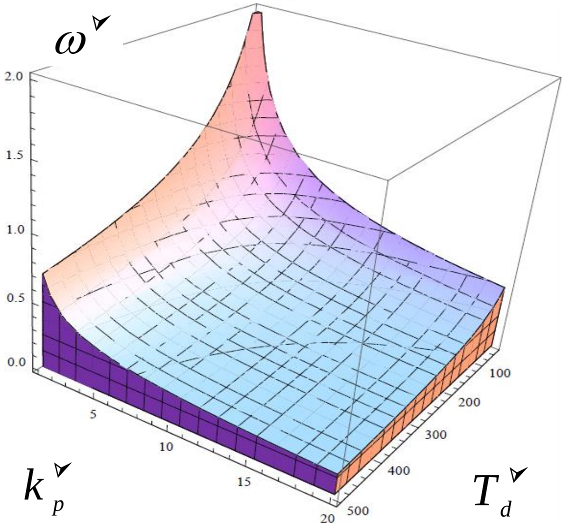
$\check{\mathcal{B}}_{PI}^{PP}\left\{\check{\omega},\check{k}_p,\check{T}_i\right\}$



Фиг.11.

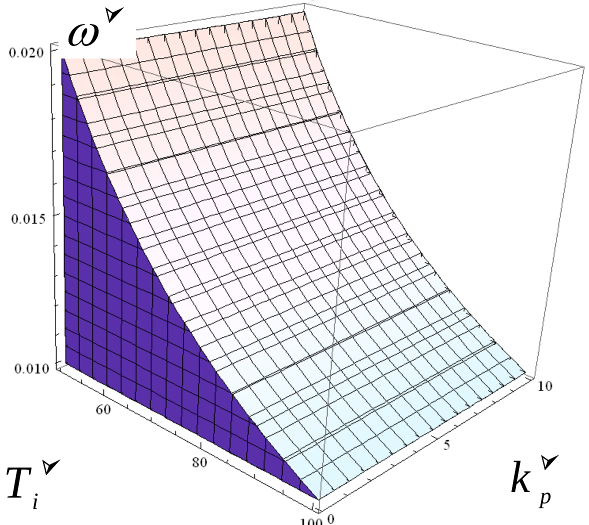
$\check{\mathcal{B}}_{PID}^{PP}\left\{\check{\omega},\check{k}_p,\check{T}_i,\check{T}_d\right\},\left(\check{T}_d = const\right)$





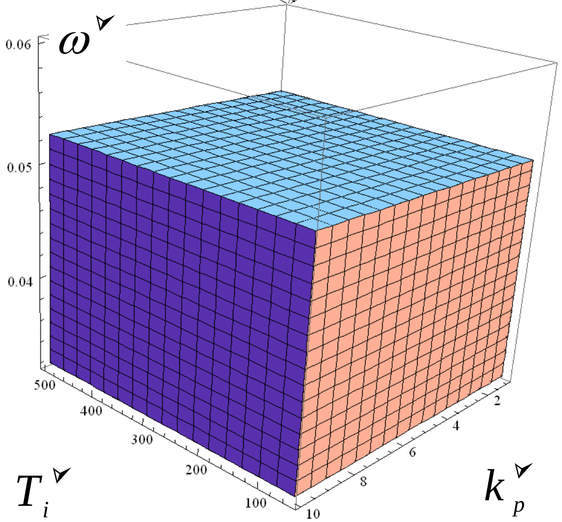
$\check{\mathcal{B}}_{PD}^{IP}\left\{\check{\omega},\check{k}_p,\check{T}_d\right\}$

Фиг.12.



$\check{\mathcal{B}}_{PI}^{IP}\left\{\check{\omega},\check{k}_p,\check{T}_i\right\}$

Фиг.13.

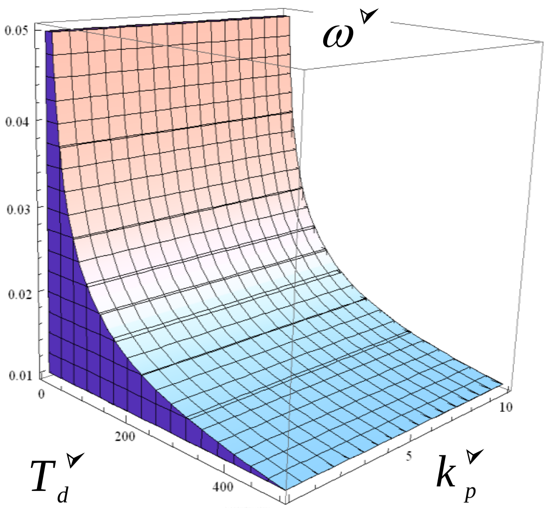


$\check{\mathcal{B}}_{PID}^{IP}\left\{\check{\omega},\check{k}_p,\check{T}_i,\check{T}_d\right\},\left(\check{T}_d = const\right)$

Фиг.14.

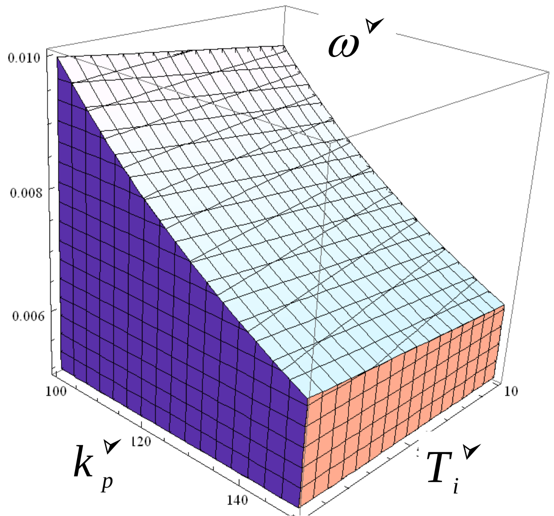
Фиг.15.

$\check{\mathcal{B}}^{AA}_{PD}\left\{\check{\omega},\check{k}_p,\check{T}_d\right\}$



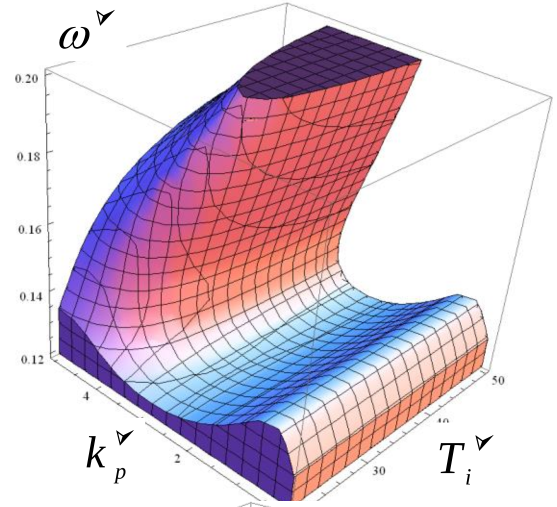
Фиг.16.

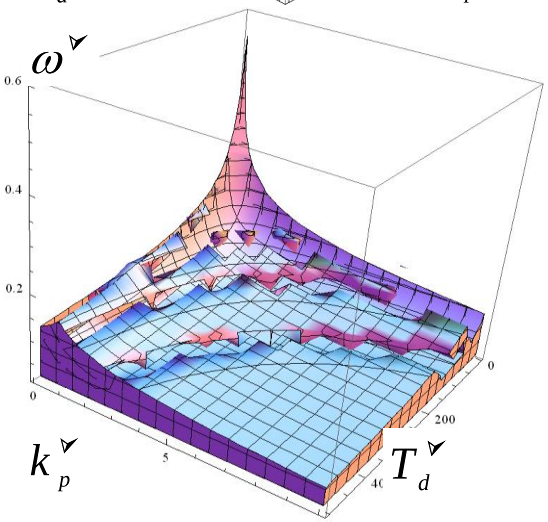
$\check{\mathcal{B}}^{AA}_{PI}\left\{\check{\omega},\check{k}_p,\check{T}_i\right\}$



Фиг.17.

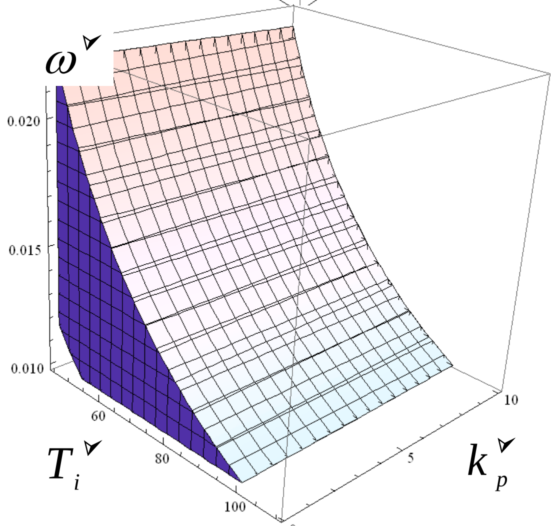
$\check{\mathcal{B}}^{AA}_{PID}\left\{\check{\omega},\check{k}_p,\check{T}_i,\check{T}_d\right\},\left(\check{T}_d = const\right)$





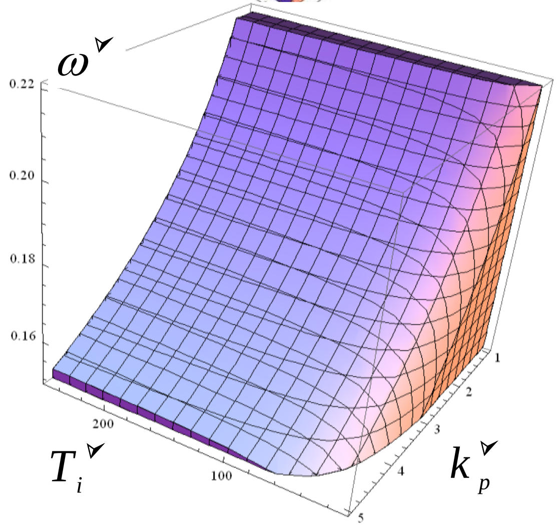
$\check{\mathcal{B}}^{IA}_{PD}\left\{\check{\omega},\check{k}_p,\check{T}_d\right\}$

Фиг.18.



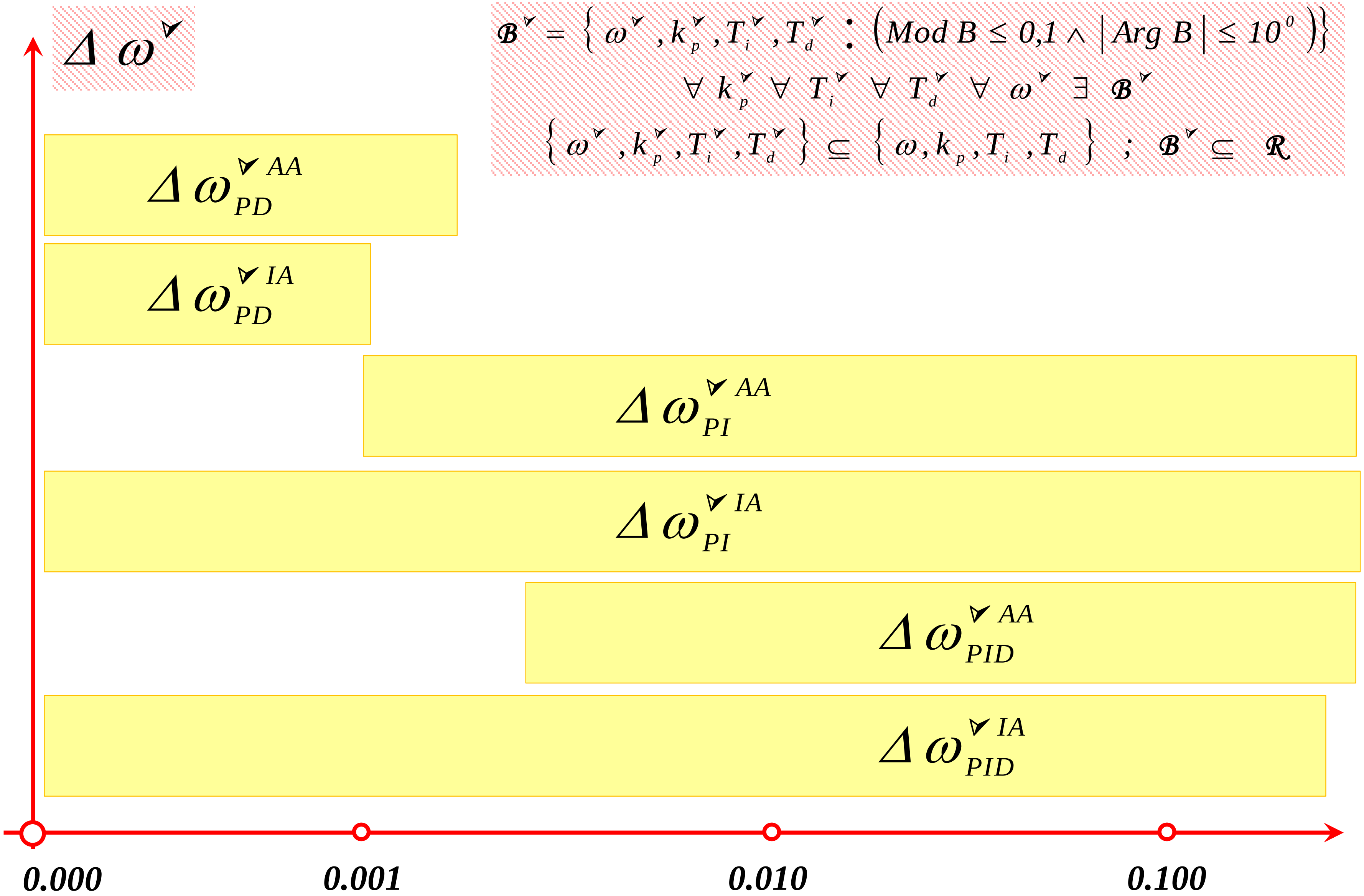
$\check{\mathcal{B}}^{IA}_{PI}\left\{\check{\omega},\check{k}_p,\check{T}_i\right\}$

Фиг.19.



$\check{\mathcal{B}}^{IA}_{PID}\left\{\check{\omega},\check{k}_p,\check{T}_i,\check{T}_d\right\},\left(\check{T}_d = const\right)$

Фиг.20.


$\Delta\,\omega^{\vee}$
$\mathcal{B}^{\vee} = \{\omega^{\vee}, k_p^{\vee}, T_i^{\vee}, T_d^{\vee} : (Mod\,B \leq 0{,}1 \wedge |Arg\,B| \leq 10^0)\}$
$\forall\, k_p^{\vee}\ \forall\, T_i^{\vee}\ \forall\, T_d^{\vee}\ \forall\, \omega^{\vee}\ \exists\ \mathcal{B}^{\vee}$
$\{\omega^{\vee}, k_p^{\vee}, T_i^{\vee}, T_d^{\vee}\} \subseteq \{\omega, k_p, T_i, T_d\}\ ;\ \mathcal{B}^{\vee} \subseteq \mathcal{R}$
$\Delta\,\omega_{PD}^{\vee AA}$
$\Delta\,\omega_{PD}^{\vee IA}$
$\Delta\,\omega_{PI}^{\vee AA}$
$\Delta\,\omega_{PI}^{\vee IA}$
$\Delta\,\omega_{PID}^{\vee AA}$
$\Delta\,\omega_{PID}^{\vee IA}$
0.000
0.001
0.010
0.100
$\omega^{\vee}, rad/s$

Фиг.21.

ЗАКЛЮЧЕНИЕ И ИЗВОДИ

Новото и оригинално в настоящата работа са предложените:

● систематизация, обобщения и аналитични дефиниции за параметризацията на показатели (***OHP***) на характеристиката на динамични системи;

● подход, метод и алгоритъм за параметризация на динамични системи;

● числени примери, извод на символни аналитични описания на характеристиките на конкретни динамични системи; анализ на системите;

● определяне на аналитични описания на съставящи на характеристиките на промишлени динамични системи в качеството им на техни свойства (***OHP***);

● функционална параметризация на съставящите с независимите променливи на динамични системи за конкретни числени примери;

● символна визуализация на резултатите от параметризацията с технологиите и инструментариума на ***2D-параметрично-контурен*** плот и на ***3D-параметрично-контурен*** плот като алгоритми за параметризация на функции на повече от две независими променливи, сравнителен анализ на възможностите им.

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ИНФРАСТРУКТУРНИ УСЛУГИ ЗА ОБЛАЧНИ ИЗЧИСЛЕНИЯ В МРЕЖАТА ЗА РАДИОДОСТЪП

Евелина Пенчева, Пенчо Пенчев, Мирослав Славов

Резюме: *Multi-access Edge Computing (MEC) е технология, която се отнася до разпределена мобилна система за облачни изчисления, в която компютърните ресурси се инсталират в рамките на мрежата за радиодостъп. В работата е представен метод за проектиране на инфраструктурни уеб услуги, които осигуряват среда за комуникация на MEC платформата с приложенията. Дефинирани са приложни програмни интерфейси, които могат да се използват за мениджмънт на натоварването и работоспособността. Функционалността на уеб услугите се основава на изискванията за интегритет на мрежовата функционалност.*

Ключови думи: *уеб услуги, мениджмънт на интегритета, мениджмънт на работоспособността*

INFRASTRUCTURE SERVICES FOR MULTI-ACCESS EDGE COMPUTING

Evelina Pencheva, Pencho Penchev, Miroslav Slavov

Abstract: *Multi-access Edge Computing (MEC) is a technology that refers to a distributed mobile cloud computing system in which computer resources are installed within the radio access network. The paper presents a design method for infrastructure web services, which provide an environment for communication between the MEC platform and applications. Application programming interfaces that can be used for load management and performance management are defined. The functionality of web services is based on the requirements for integrity of network functionality.*

Key words: *web services, integrity management, performance management*

1. ВЪВЕДЕНИЕ

Multi-access Edge Computing (MEC) предоставя възможностите на облачните изчисления като съхранение и обработка на данни в мрежата за радиодостъп. MEC поддържа множество технологии за достъп като Wi-Fi, 3G, 4G, 5G, Wi-Max и Bluetooth [1]. MEC може да обработва и оптимизира големи обеми от данни, събрани от мобилни устройства, преди да ги прехвърли в облака, което предотвратява загубата на ресурси. Това е технология, подходяща за различни приложения в реално време. Услугите на MEC имат директен достъп до локална контекстна информация, базирана на местоположението на потребителя, усло-

ISSN 1311-0829

вията в мрежата за достъп, информация за поведението на мобилните потребители и др. [2], [3].

MEC се отнася до разпределена мобилна система за облачни изчисления, в която компютърните ресурси се инсталират в рамките на мрежата за радиодостъп, близо до края на мобилните устройства на Интернет [4]. Мобилните крайни хостове са компютърно оборудване, инсталирано в или близо до базовите станции. За разлика от централизираните облачни сървъри или мобилните устройства, които комуникират директно помежду си, MEC се управлява локално от мрежовия оператор. Общите компютърни ресурси в мобилните крайни хостове са виртуализирани и са изложени чрез приложни програмни интерфейси (API), така че да са достъпни както за приложения на потребители, така и за оператори [5].

MEC предоставя три вида мидълуерни услуги:

- Инфраструктурни услуги, които позволяват на приложенията да комуникират с услугите на платформата посредством API и да откриват услуги, налични в MEC сървъра;
- Услуги за информация за радиомрежата, които предоставят на оторизирани приложения информация за радиомрежата от ниско ниво;
- Функция за разтоварване на трафика, която приоритизира трафика и маршрутизира избрания поток от данни на базата на правила към и от приложения, които са оторизирани да приемат данните.

Съвременните научни изследвания са фокусирани върху уникалните предизвикателства пред проектирането на MEC платформи. Изследванията описват таксономията на MEC и ключовите атрибути на технологията. В [6] изчисленията в края на мрежата са представени като нововъзникваща технология, която позволява нови възможности, включително опростяване на управлението на облака, високодостъпни услуги, мащабируемост чрез анализ в края на мрежата, прилагане на политиката за защита на личните данни и т.н. В [7]. MEC е описана като ключов елемент от 5G системи с акцент върху функционалността и архитектурата на MEC, като са обсъдени някои примерни случаи на използване. В [8] са представени ключови случаи и сценарии за използване на MEC и е обсъдена стандартизацията на MEC. Авторите се фокусират върху технически изследвания, които водят до изчислителното разтоварване на мрежата към MEC. Авторите на [9] представят текущите тенденции в MEC, проблемите и предизвикателствата към сигурността и свързаните с тях научните изследвания. В [10] авторите правят изчерпателен преглед на най-съвременните изследователски усилия в мобилните крайни мрежи, като се съсредоточават върху ключовите възможности на мобилните крайни мрежи, като облачната технология, виртуализация на мрежови функции и интелигентни устройства.

В тази статия е предложен метод за проектиране на инфраструктурни MEC услуги за мениджмънт на натоварването и работоспособността.

Изложението е структурирано както следва. В точка 2 е представен метод за проектиране на API за мениджмънт на натоварването, а в точка 3 – метод за проектиране на API за мениджмънт на работоспособността. Методите са илюстрирани с типични случаи на използване интерфейсите, описанието на самите интерфейси и модели на състоянията на съответните ресурси. Заключението обобщава основните приноси.

2. УЕБ УСЛУГА ЗА МЕНИДЖМЪНТ НА НАТОВАРВАНЕТО

Функционалността на мрежата не трябва да бъде компрометирана от приложение, което прави твърде голям брой заявки и обратно, MEC платформата не трябва да се претоварва от прекалено голям брой заявки, генерирани от приложения.

Уеб услугата Load Management позволява на MEC платформата да наблюдава и управлява натоварването на приложението в съответствие с правила за управление на натоварването. Политиката за управление на натоварването определя правилата за управление на товара, които трябва да се следват.

Фиг.1 показва диаграма на последователности за управление на натоварването на приложения от MEC платформа.

1-2. MEC платформа извиква операция *startLoadNotification*, за да се абонира да получава известия за промените в нивото на натоварване на приложението.

3-4. Приложението информира MEC платформата за текущото ниво на натоварване, като извиква операцията *reportLoad*. Може да се определят например три нива на натоварване. Когато приложението не е претоварено, нивото му на натоварване е "0". Когато приложението е претоварено, нивото му на натоварване е "1". При ниво на натоварване "2" приложението е силно претоварено.

5-6. MEC платформата установява промяна на натоварването, което води до състояние на претоварване и извиква операцията *holdNotification*, за да изиска от приложението да задържи изпращането на всички известия.

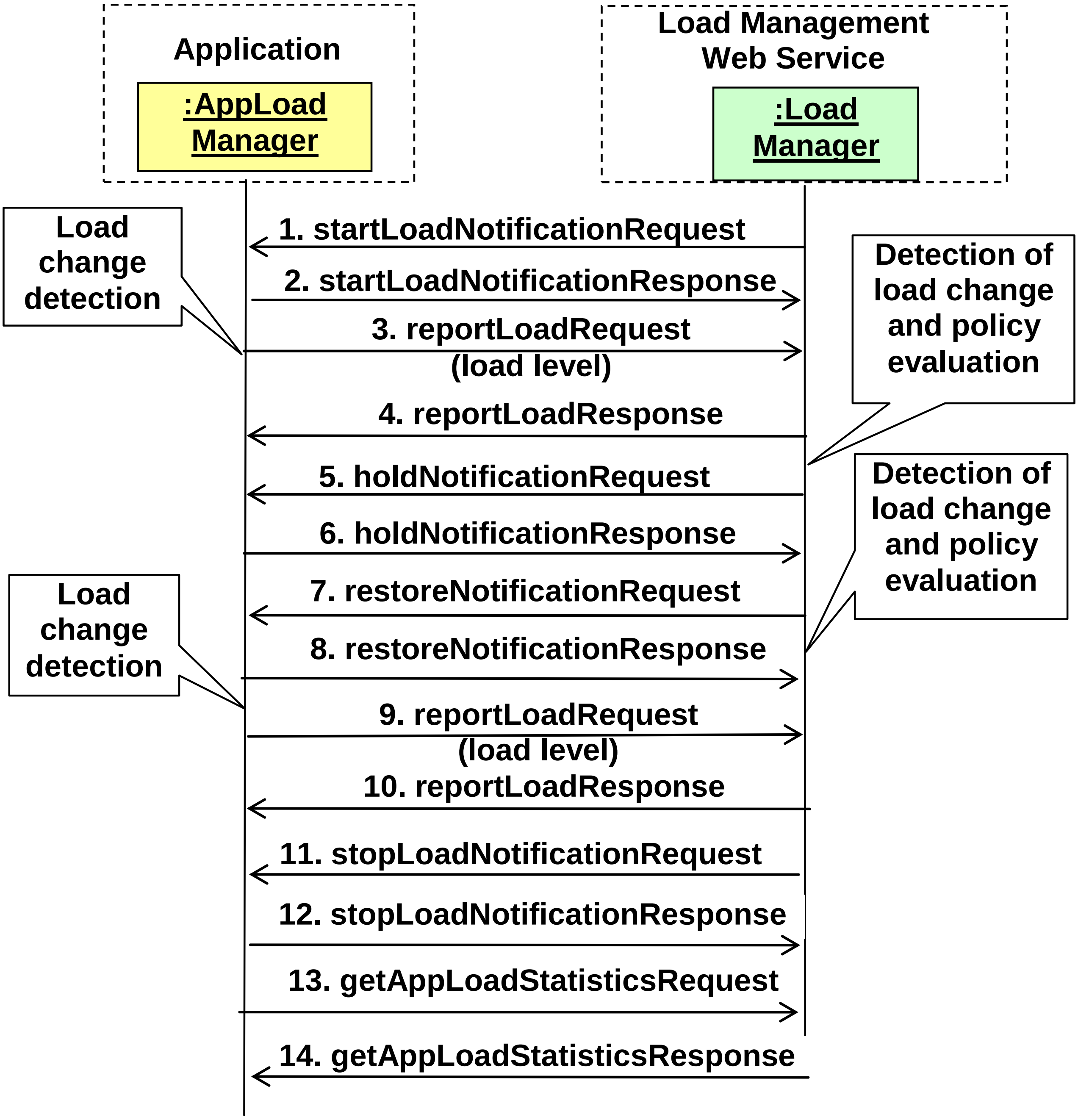
7-8. След период на задържане на известия за натоварването на приложението, то се справя с временното състояние на претоварване. MEC платформата извиква операция *restoreNotification*, която изисква приложението да възстанови известията за нивото на натоварване.

9-10. Приложението информира MEC платформата за текущото си ниво на натоварване.

11-12. MEC платформата извиква операция *stopLoadNotification*, за да прекрати абонамента за известяване за промени в нивото на натоварване на приложението.

13-14. Приложението извиква операцията *getAppLoadStatistics*, за да поиска от MEC платформата да предостави запис на статистически данни за натоварване за приложението.

Фиг.2 показва интерфейсите на уеб услугата Load Management и поддържаните от тях операции.



Фиг.1. Мениджмънт на натоварването на приложение.

Интерфейсът *AppLoadManager* осигурява следните операции.

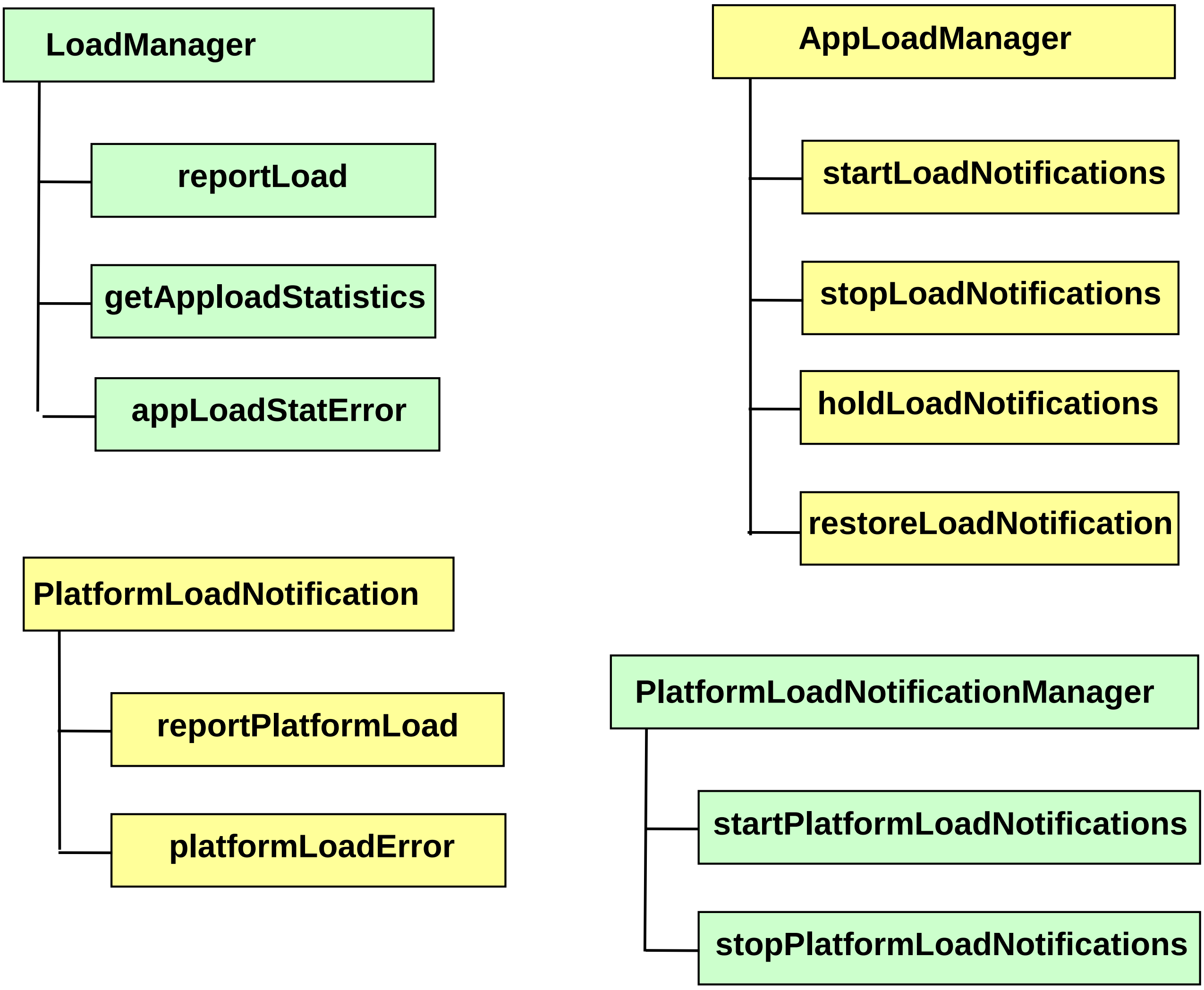
Операцията *startLoadNotification* се използва от MEC платформата за абонамент за известия за промени в нивото на натоварване на приложението.

Операцията *stopLoadNotification* се използва за прекратяване на абонамента за известия за промени в нивото на натоварване на приложението.

Операцията *holdNotification* се използва от MEC платформата за временно задържане на известията от приложението. MEC платформата използва операция *restoreNotification*, за да поиска от приложението да възстанови изпращането на известия.

Операцията *appLoadStatisctics* се използва от MEC платформата, за да изпрати статистика на приложението за нивото си на натоварване.

Операцията *appLoadStatError* се използва от MEC платформата, за да информира приложението, че статистическите данни за натоварването на платформата не могат да бъдат изпратени поради грешка.



Фиг.2. Интерфейси на услугата Load Management и поддържаните операции.

Интерфейсът *Load Manager* осигурява следните операции.

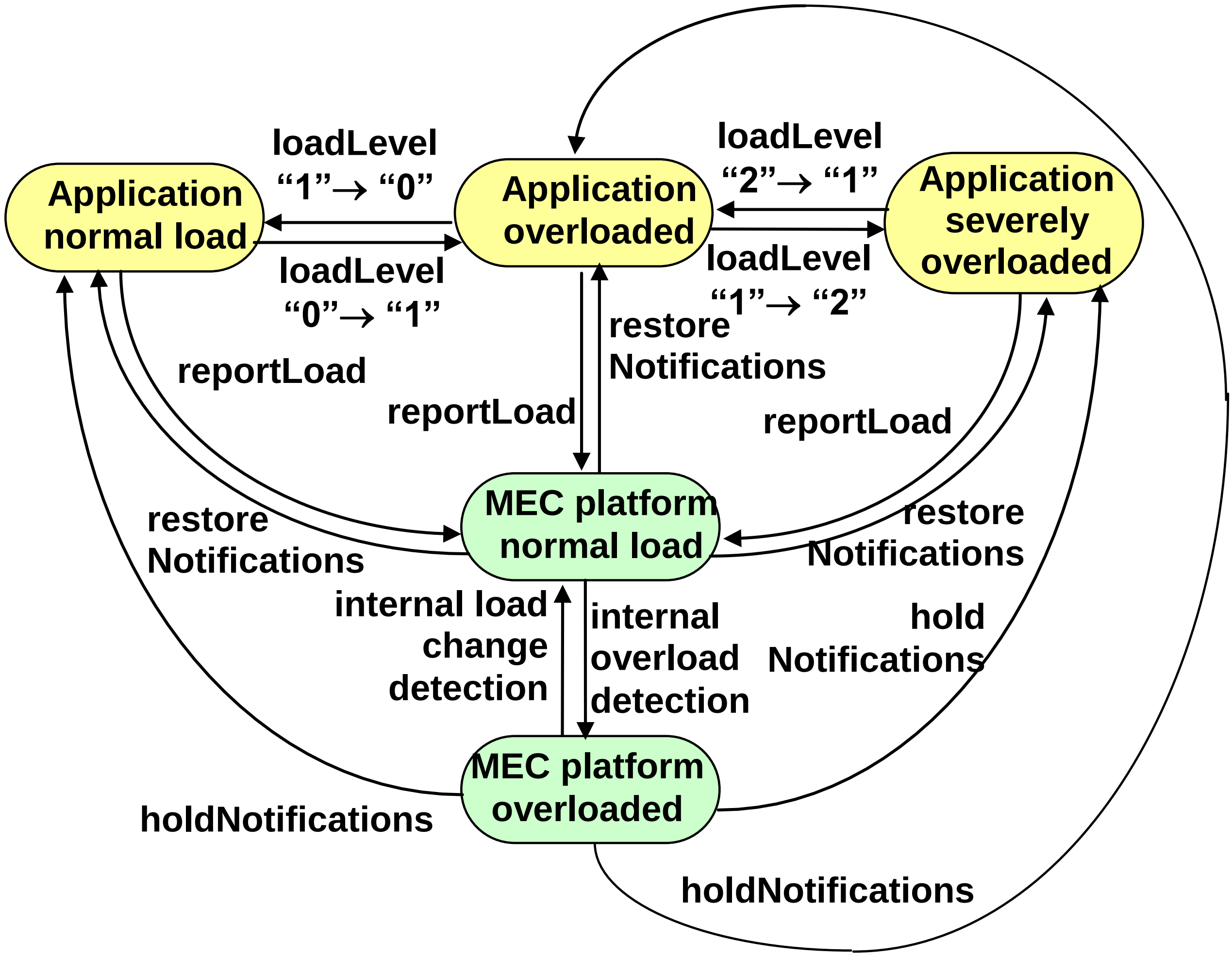
Приложението използва функцията *reportLoad* за докладване на изменения в нивото на натоварване.

Операцията *getAppLoadStatistics* се използва от приложението, за да поиска от MEC платформа за статистически данни за натоварване на приложението.

Интерфейсът *PlatformLoadNotificationManager* се поддържа от платформата и осигурява операции, с които приложението може да се абонира да получава известия за натоварването на MEC платформата.

MEC платформата известява приложението за нивото на натоварването си като извиква операциите, поддържани от интерфейса *PlatformLoadNotification*.

Фиг.3 показва диаграма на състоянието, представяща вътрешните нива на натоварване на MEC платформата и на приложението.



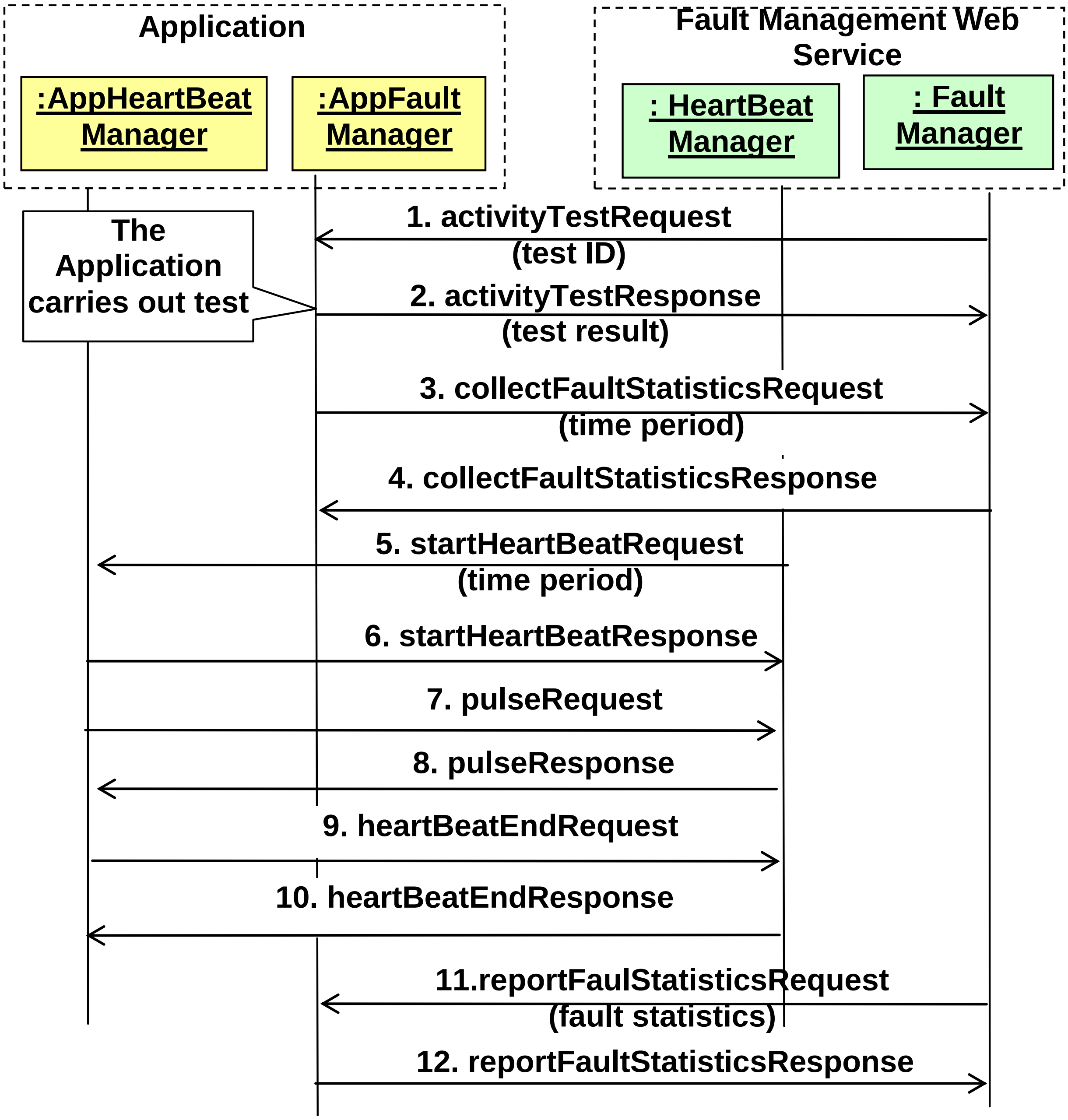
Фиг.3. Състояния на натоварването на MEC платформата и приложението.

3. УЕБ УСЛУГА ЗА МЕНИДЖМЪНТ НА НЕИЗПРАВНОСТИ

Уеб услугата Fault Management позволява на MEC платформата да следи оперативното състояние на приложението.

MEC платформата може да се регистрира, за да получава периодични известия (heart beatings) от приложението за проверка на състоянието му на активност.

На фиг.4 показана диаграма на последователности за мениджмънт на неизправности на приложения.



Фиг.4. Периодично докладване на активността на приложението.

1. MEC платформата извиква операция *activityTest*, за да провери дали приложението е работоспособно. При приемане на заявката приложението трябва да се самотества, за да определи статуса си на активност.
2. Приложението връща резултата от самотестването.

3-4. Приложението извиква операцията *collectFaultStatistics*, за да изиска от MEC платформата да събира статистически данни за неизправности.

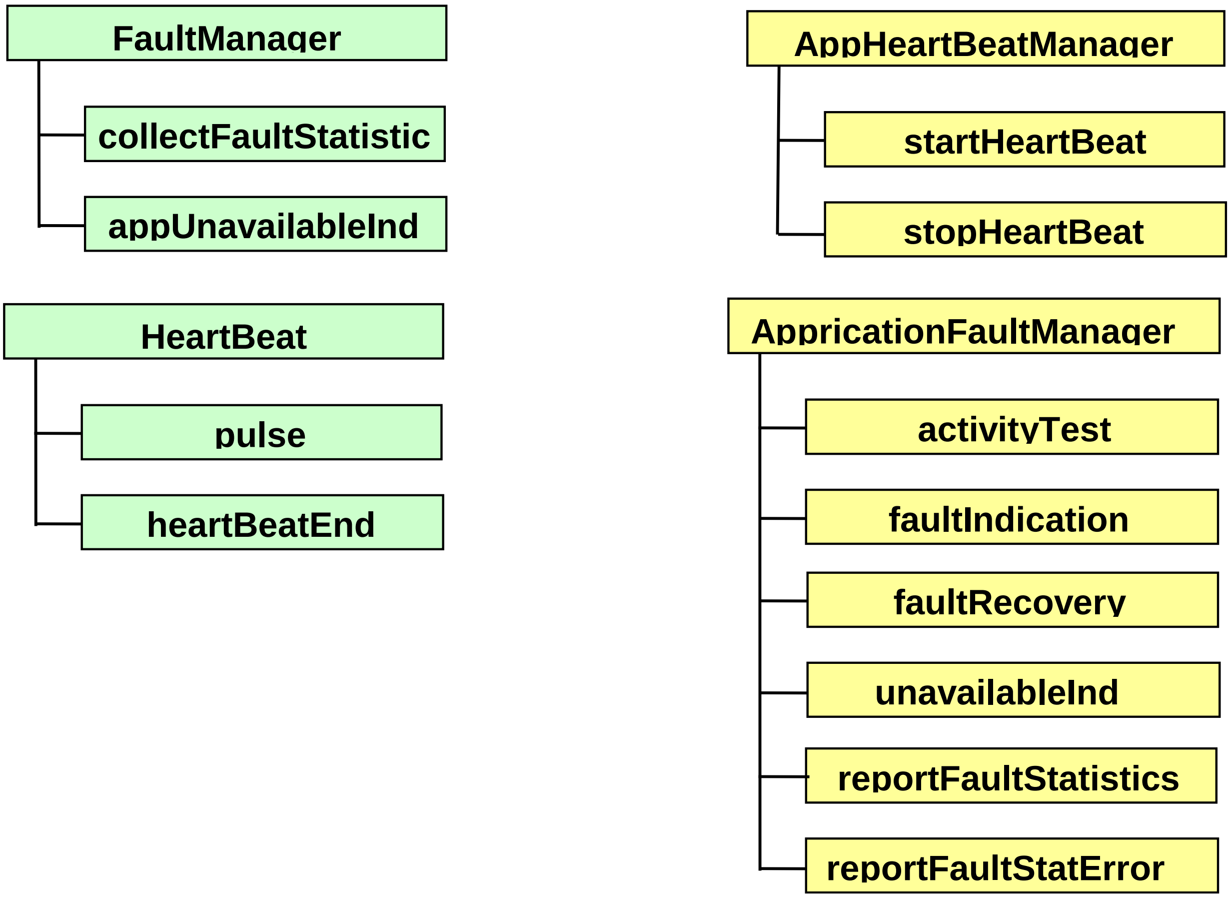
5-6. MEC платформата инструктира приложението да започне да изпраща „сърдечния си ритъм” през определен интервал.

7-8. Приложението изпраща известяване на MEC платформата.

9-10. Приложението информира платформата MEC, че срокът за изпращане на известия е приключил.

11-12. MEC платформата осигурява натрупаната статистика за неизправности.

Фиг.5 обобщава интерфейсите и поддържаните операции на уеб услугата Fault Management.



Фиг.5. Интерфейсите и поддържани операции на уеб услугата Fault Management.

Интерфейсът *AppFaultManager* се използва от MEC платформата за наблюдение на активността на приложение. Операцията *activityTest* изисква от приложението да направи самотестване за активност. Операцията *faultIndication* известява приложението за неизправност. Когато откритата неизправност е отстранена, MEC платформата извиква операция *faultRecovery*, за да извести на приложението. MEC платформа извиква операция *unavailableInd*, за да уведоми приложението, че вече не е налична. Извиквайки операцията *reportFaultStatistics*, MEC платформата докладва на приложението статистически данни за неизправности в платформата. Операцията *reportFaultStatError* се извиква, когато MEC платформата не може да предостави статистически данни за неизправности на приложението.

Интерфесът *FaultManager* се използва от приложението, за да информира MEC платформата за събития, които засягат целостта на платформата MEC и да изискват информация за интегритета на системата.

Операцията *collectFaultStatistics* се извиква от приложението, за да изиска от MEC платформата събирането на статистика за грешки за определен период от време. Приложението извиква операция *appUnavailableInd*, за да информира платформата MEC за състоянието му на недостъпност като посочи причината.

Интерфейсът *AppHeartBeatManager* позволява иницииране на докладване на „сърдечния пулс" на приложението от MEC платформата. Платформата извиква операция *startHeartBeat,* за да инструктира приложението да започне периодични докладвания. Платформата извиква операцията *stopHeartBeat*, за да инструктира приложението да престане да изпраща периодични докладвания.

Интерфейсът *HeartBeat* се използва от приложението за изпращане на периодични докладвания за наличност. Приложението операция *pulse,* за да докладва на платформата наличността си. Когато изтече срокът за изпращане на „сърдечния ритъм", приложението извиква операцията *heartBeatEnd*, за да информира MEC платформата, че периодът на наблюдение на наличността на приложението е приключил.

4. ЗАКЛЮЧЕНИЕ

Съгласно изискванията към MEC системи, MEC платформата трябва да осигурява и функции, които не се поддържат от съществуващи приложни програмни интерфейси. В тази публикация е предложен метод за дефиниране на API за инфраструктурни услуги. Дефинирани са две инфраструктурни услуги, посредством които, приложенията могат да получат достъп до MEC услуги и други приложения, изпълняващи се на една и съща MEC платформа.

Уеб услугата Load Management осигурява функционалност, която защитава MEC платформата от компрометиране от приложения, които правят прекалено голям брой заявки. Това се постига чрез функции за управление на натоварването, които позволяват наблюдение и контрол на нивата на натоварване. Уеб услугата Fault Management осигурява функции за мениджмънт на неизправности, които включват средства за наблюдение на експлоатационното състояние на мидъуеърните услуги и приложения.

Предложеният метод за дефиниране на приложни програмни интерфейси е гъвкав и отворен за понататъшни разширения към решения от тип RESTful с подходящи адаптери.

Предложените MEC инфраструктурни уеб услуги позволяват на мрежовия оператор да управлява по-ефективно жизнения цикъл на приложенията, включващ инсталиране, стартиране, спиране и деинсталиране.

БЛАГОДАРНОСТИ

Представените изследвания са в рамките на научноизследователски договор ДН 07/10, финансиран от Фонд „Научни изследвания", МОН, Република България.

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Годишник на Технически Университет - София, том 67, книга 3, 2017
Proceedings of the Technical University of Sofia, Volume 67, Issue 3, 2017

ПРОЕКТИРАНЕ НА МОБИЛНА СИСТЕМА ЗА МОНИТОРИНГ НА ШУМ

Марин Б. Маринов

Резюме: *Повишените нива на шума (т.нар. шумово замърсяване) в градските среди е често срещан проблем, който сериозно засяга здравето и качеството на живот на хората. Ето защо много изследвания са посветени на измерване нивата на шума в околната среда. В настоящата статия се описва проектирането и създаването на прототип на мобилна безжична система за измерване на тези нива. Системата използва платформата Raspberry Pi 3 и икономически ефективни електронни компоненти. Ресурсите на Raspberry Pi и нейната мащабируема архитектура позволяват измерването на шумовите нива да се извършва едновременно с изчисляването на основни параметри на шума. Изследването на функционалността на възлите при дългосрочни тестове показва, че предлаганият подход осигурява добра основа за мониторинг и създаване на карти за нивата на шума, наричани „шумови карти“.*

Контролни думи: *шум в околната среда, измерване на звуковото налягане, постоянното еквивалентно ниво на звуково налягане, Raspberry Pi*

DESIGN OF MOBILE NOISE MONITORING SYSTEM

Marin B. Marinov

Abstract: *Noise pollution in urban environments is a common problem that affects people's health and quality of life. It is therefore one of the main issues addressed in many studies of environmental noise level measurement. This paper describes the design and rapid prototyping of a mobile wireless environmental noise monitoring system based on a Raspberry Pi 3 platform and cost-effective electronic components. Raspberry Pi resources and its scalable architecture allow acoustic level measurement to be carried out simultaneously with the computation of the environmental noise levels. Evaluation of nodes functionality in long-term measurements shows that the proposed approach provides a good foundation for noise monitoring and mapping.*

***Key words**: environmental noise, sound pressure measurement, equivalent continuous sound pressure level, Raspberry Pi*

ISSN 1311-0829

1. INTRODUCTION AND MOTIVATION

Nowadays, noise exposure monitoring and its reduction are among the main concerns for citizens, politicians, administrations, and technical scientific bodies. Directive 2002/49/EC (Environmental Noise Directive, END) is yet another attempt to harmonize the policies and technical approaches adopted by European Member States in an effort to address noise exposure reduction issues. In agreement with the Directive noise maps of relevant environmental noise sources in large population centers have been produced and this has made it possible to evaluate noise levels in major European agglomerations.

The environmental quality loss, and its impact, especially in large cities, on the health and welfare of the people is one of the greatest threats of our century. Noise in urban environments has increased significantly over the last decades, due to a growing urban development. With growing population density a greater number of people experience noise annoyance and this affects their daily life, sleep and work. Recognizing this as a major issue, the European Commission adopted a directive, which requires that Member States gather real data on noise exposure (noise maps), and develop noise management action plans for agglomerations with more than 100 000 residents, with major roads, major railways and major airports [1, 2].

Coronary heart problems and stroke are the source of around 10 000 cases of premature death. Moreover, nearly 90 % of those noise related tolls are linked to road traffic noise. It should be mentioned that a great number of countries do not provide thorough and precise data which means that the number of deaths is surely miscalculated. Using data provided by countries for the period 2006 – 2012 average noise exposure (i.e. L_{den} over 55 dB and L_{night} over 50 dB) in chosen city areas remained almost constant. Information provided by countries can be seen in Fig. 1.

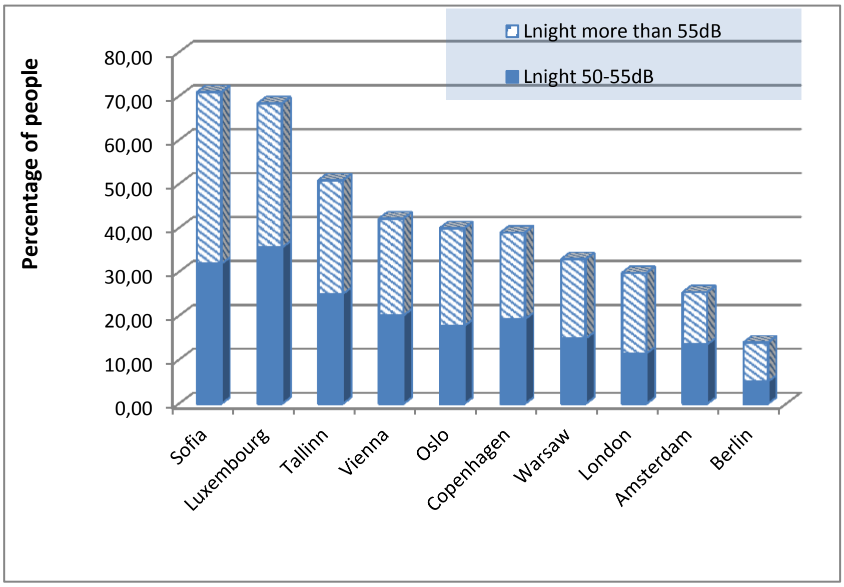

Fig. 1. Number of people subject to nighttime noise from road traffic over 50dB in several capital cities, 2012 [4].

It shows that a large percentage of people in several capitals cities throughout Europe are exposed to high risk levels of road traffic noise. Unfortunately, Sofia with about 70 % of its residents exposed to nighttime noise from road traffic of over 50 dB holds a 'leading place' in this unpleasant statistics [3].

Traditional approaches of performing noise measurements are time consuming and expensive due to the personnel and measuring equipment costs. Noise maps are created [5] on the basis of the measurements and by means of numerical tools and propagation models. The actual European Commission recommendations are for higher time and space granularity of the noise data. In this situation, mobile noise monitoring systems with wireless connectivity/communication can provide an attractive alternative for overcoming the drawbacks of the current noise data acquisition techniques [6]. There is high demand for devices that provide online and near real-time noise level measurements to improve and fine-tune the information from the noise maps. The present research is primarily aimed at the development and practical use of wireless mobile devices for noise monitoring with scalable architecture as well as at carrying out an analysis of the suitability of usage of off-the-shelf hardware alternatives.

2. ACOUSTICAL DESCRIPTORS

Acoustic measurement is a process influenced by static and dynamic variables. Noise constantly changes in the time and space domain.

Human ears are sensitive organs, which can hear sound power in a wide range of up to 13 decimal magnitudes [7]. The sound power level is defined as a logarithm scale, denoted as decibels, and can be calculated by

$$L_p = 10\log\left(\frac{p_{rms}^2(t)}{p_{ref}^2}\right) = 20log\left(\frac{p_{rms}}{p_{ref}}\right)\ dB \tag{1}$$

where p_{rms} and p_{ref} are RMS (Root Mean Square) sound pressure and reference sound pressure, respectively with $p_{ref} = 20$µPa.

The *equivalent sound pressure level* L_{eq} is the most widely used single index indicator for evaluation of exposure to noise. It can be used for rating sounds with levels varying over time. L_{eq}, T is the average sound pressure level for a suitable period T. The Equivalent Sound Level is a measure of exposure due to the accumulation of sound levels over a particular period. The applicable period should always be identified when using this metric. L_{eq}, T may be thought of as a constant sound level over a period of interest that contains as much sound energy as the actual varying level. It is a way of assigning a single parameter to a time-varying sound level.

For most people the normal frequency range of hearing extends from about 20 Hz to about 10 000 - 15 000 Hz. People respond to sound most readily when the dominant frequency is within the range of normal conversation, typically around 1 000 to 2 000 Hz. The acoustical community has defined several "filters", which approximate this sensitivity of the ear for better assessment of the relative loudness of various sounds made up of many different frequencies. The “A-weighting” filter does this best for most environmental noise sources. The A-filter weighting curve is defined as in ANSI S1.42.2001 [8] and the attenuation limits as in IEC 61672-1:2002 [9]. The A-filter is a frequency-selective filter. It passes the frequencies in the frequency range between 1 and 4 kHz, to which the human ear is most sensitive and suppresses very high and very low frequencies.

If pressure levels are weighted by the A-weighting curve the equivalent continuous sound pressure level(L_{Aeq}, T) is obtained as follows:

$$L_{Aeq,T} = 10\log\frac{1}{T}\int_0^T 10^{\frac{L_A(t)}{10}}dt = 10\log\frac{1}{T}\int_0^T 10^{\frac{p^2_{Arms}(t)}{p^2_{ref}}}dt, \qquad (2)$$

where $L_A(t)$ is the instantaneous sound level A-weighted, $p_{Arms}(t)$ is the sound pressure measured A-weighting frequency filter. $L_{Aeq,T}$ is one of the most widely used descriptors in evaluating noise in urban environments, roads, railways, and industry [10].

Directive 2002/49/EC prescribed the use of indicator L_{den}, which represents the day-evening-night level in dB:

$$L_{den} = 10\log\frac{1}{24}\left(12\cdot 10^{\frac{L_{day}}{10}} + 4\cdot 10^{\frac{L_{evening+5}}{10}} + 8\cdot 10^{\frac{L_{night+10}}{10}}\right). \qquad (3)$$

where L_{day}, $L_{evening}$ and L_{night} are the A-weighted long-term average sound levels as determined in ISO 1996-2: 1987, defined over all daytime periods of a year.

Here the average day is 12 hours, the evening is 4 hours, and the night is 8 hours. For the purpose of noise mapping, the European Community Member States must provide noise pollution data in terms of the L_{den} and L_{night} indicators.

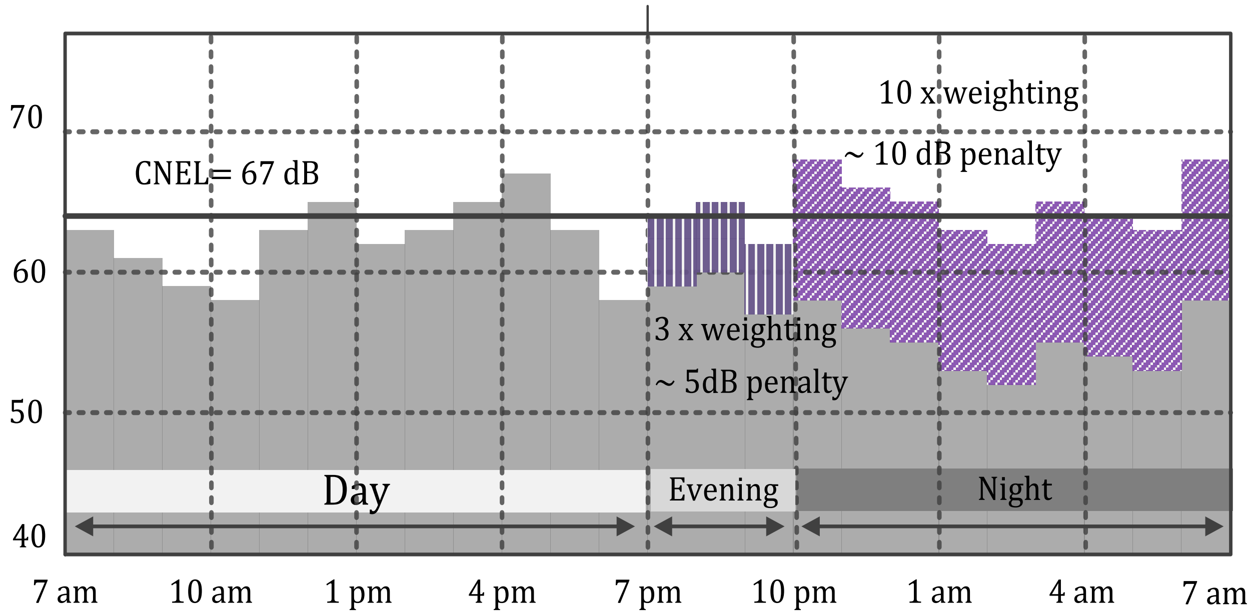

Fig. 2. L_{den} calculation [11].

The L_{den}, (day-evening-night level) parameter is the L_{eq} measured over a 24-hour period with an added penalty of $10\ dB$ for the levels between 23:00 and 07:00 and a $5\ dB$ penalty added to the levels between 19:00 and 23:00 to account for people's extra noise sensitivity during these time intervals.

As L_{eq} is related to the effect of noise on people, L_{den} extrapolates this to a daily value and, in long-term measurements, to weekly, monthly or yearly data for more long-term studies [12].

3. SYSTEM DESIGN

3.1. Design requirements

For this particular design process, the following requirements have to be met:

- Measurement, storage and calculation of basic environmental noise parameters are to be performed.
- Computing power to perform on-board calculations, scalable architecture that supports easy expansions with peripherals e.g. sound cards, microphones, environmental parameter sensors are to be provided.
- The measurement accuracy should comply with the requirements of IEC 61672 for Class 2 Sound level meters [9].
- High reliability and availability of the device for long-term measurements in (near) real-time and battery powered operation mode.
- The device should comprise off-the-shelf cost-effective components for WSN implementation and support different communication standards.
- Capability for remote status monitoring, GPS localization and software updates.
- The device should be enabled for MATLAB/LabVIEW programming environment.
- Data to be recorded: according to the sound level measurement standards the time should be within $5\ s$ any time of the actual time of the day. The time resolution for any clock should be at least $1\ s$ [13]. The monitor should work continuously and display on request the A-weighted sound pressure levels of the total sound in the form of time-series of 1 second or other preset time-averaged sound pressure levels.

3.2. Architecture

The Raspberry Pi 3 Platform

Taking into account the design requirements and the needed functionalities a scalable architecture based on the Raspberry Pi 3 (RPi) platform is proposed below (Fig. 3).

The main component of the device is the processing unit, which is also used for data acquisition and connectivity. Lately, a new computer concept has become very popular. These new devices, known as Single Board Computers (SBC), are smaller than traditional computers and distinguished for being more economical and affordable. The SBC has already proven its computing power and its scalability for a variety of projects. Different types of SBCs with different features of connectivity, computing power, size or energy usage are available on the market. Arduino, Raspberry Pi, BeagleBone, are widely used in the field of noise monitoring [14].

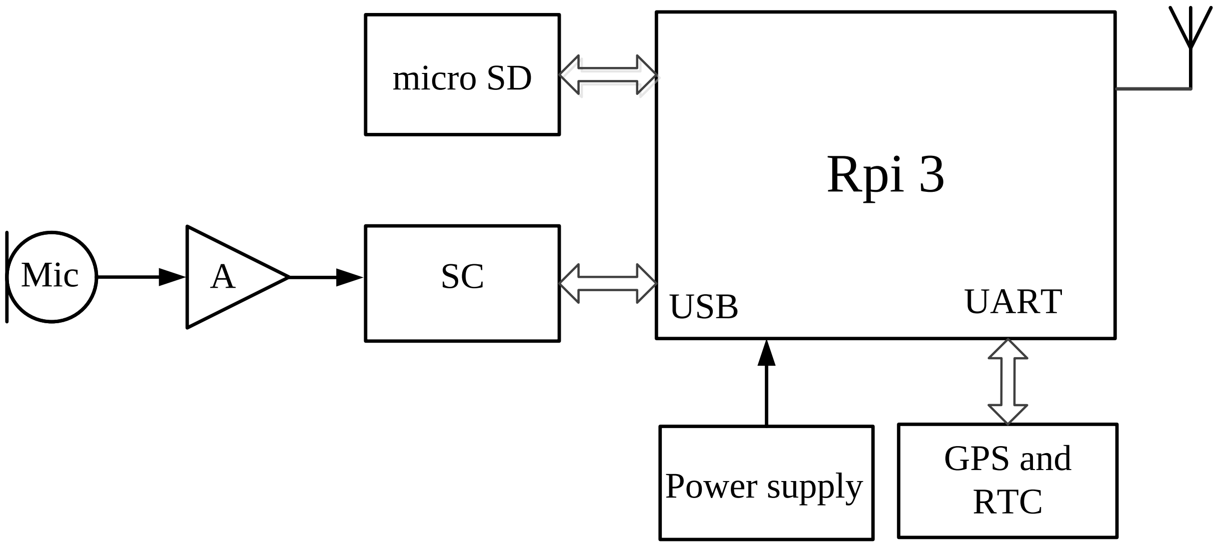

Fig. 3. System architecture.

In order to meet the set requirements, the design of the noise monitoring device was based on the platform Raspberry Pi 3 Model B single board computer. It has the following characteristics: it consists of the Broadcom BCM2837 System on a Chip, including 4 cores ARM Cortex-A53, running on a 1.2 GHz processor, a Graphic Processing Unit (GPU), 1 GB of RAM and with a microSD memory card slot.

The Raspberry Pi platform has a number of advantages such as good computing power, high versatility and that it is enabled for MATLAB/LabVIEW programming environment. The low power consumption and the price allow for the design of numerous devices based on this platform. Those qualities, together with the continuous hardware upgrades, made the Raspberry Pi the selected option for the prototype development.

Raspberry Pi has various options for data transfer - mainly via Ethernet, Wi-Fi or Bluetooth connectivity. The most appropriate alternative is the use of the IEEE 802.11n-based Wi-Fi built-in chip. The module operates at 2.4 GHz and provides up to 150 $Mbit/s$ transfer rates. The distance of the station to the access point can be up to 300 m but this strongly depends on the terrain and surrounding background radio noise. In this particular case, tests with a direct visibility of up to 50 m were conducted.

In cases when the wireless connection has a low performance due to interference from nearby devices or neighboring radio signals, an Ethernet cable is to be used. In case of a cable network, the device can be powered using a POE (Power Over Ethernet) scheme based on IEEE 802.3af.

Microphone

The microphone is the interface between the acoustic field and the noise measurement system. It reacts to the variations in the sound pressure and converts them into an electrical signal that can be interpreted by the measuring instrument. The microphone is one of the most important elements of noise measurement systems because the accuracy of measurement of the entire monitoring system depends on its characteristics. Therefore, it is important to select it carefully, taking into account several basic requirements. One of them is related to the sensitivity of the microphone represented by the ratio of the voltage received at the output and the sound pressure at the input. The necessity to model human perception requires using a microphone with a frequency range corresponding to the audibility range of the human ear.

The typical level of environmental and background noise in living areas is within the range of 30 dB to 80 dB – 85 dB. The usual sound level of highway traffic is $70 - 80$ dB and it is related to uncomfortable noise. Exposure for a longer period of time to sound levels of over 85 dB causes hearing damage. Noise peak values often reach $100 - 110$ dB, therefore, the developed sensor node is defined with a dynamic range of about 90 dB (in the range $30{-}120$ dB) for noise measurement [15].

An ADMP404 MEMS omnidirectional microphone, was installed on the RPi. The microphone is placed on a SparkFun Breakout board. It has a gain amplification of about 67 with an AD8606 operational amplifier that fully meets the bandwidth requirements for the microphone ADMP401. The amplifier's audio output will float at one half of the power supply voltage when no sound is being picked up. The amplifier produces a peak-to-peak output of about $200\,mV$ when the microphone is at a distance of $0.5\,m$ at normal conversational volume levels.

Sound Card

A solution with a sound card for converting the analogue signal from the microphone into digital form has been chosen. Using a USB sound card is a good solution because of its low cost, small size and easy connectivity. Raspberry Pi 3 has 4 USB2.0 ports and supports the software needed to easily integrate the sound card in the system. Here an external USB sound card is used (Dynamode [13]). The output format of the sound card is a 16-bit signed integer [-32768, +32767] which theoretically ensures a dynamic range of over $96\,dB$.

GPS module and real-time clock

Since the system has to keep its functionality if no internet connection is available, a real-time clock (RTC) with a rechargeable back-up battery power supply is needed. The selected Raspberry PI computer does not have a RTC, which is a drawback when organizing records and making reports about the noise measurements made. Alternatively, GPS time can be used. The selected GPS module is NEO-7M. The receiver uses a battery powered real-time clock and backup RAM.

Micro SD card

The measured acoustic parameters, the basic program, and the files required for the operating system are stored and kept on the memory card.

4. EXPERIMENTAL RESULTS

4.1 The Complete RPi node

Fig. 4 shows the complete RPi sound level node.

Fig. 4. View of the noise monitoring system.

The node is placed in a plastic enclosure with IP 67 protection with microphone tube, wind protection, GPS module and power bank. The components used in RPi node and their approximate prices are given in Table 1.

Table 1.
List of the components for the sensor node

Part	Commercial name	Price, Euro
Main board	Raspberry Pi 3	35
Microphone	ADMP 401	16
Soundcard	Dynamode USB	4
GPS Module	GPS NEO-7M	30
Enclosure	-	15
Cables + accumulators	-	25

The cost of the components is about 125 Euro. When we compare this price with the typical price for class I sound meter (1500 – 3000 Euro), this WSN application is highly competitive [6].

4.2. Power consumption

Instantaneous power consumption was measured by connecting a $1.5\ \Omega$ resistor in series to the power supply and by monitoring the voltage drop for a duration of 60 minutes. The average current consumption is approximately $510\ mA$ with enabled wireless connection and GPS and disabled HDMI, Ethernet port and other communication channels. This translates to more than 24 hours operational time with $16\ Ah$ battery pack (for accumulator pack efficiency > 80%).

5. CONCLUSION

Monitoring of noise parameters is an area of research that has been the focus of scientific attention in the last decade. In the present paper the development of a cost-effective wireless sensor node for environmental noise measurement is described. The hardware design challenges are discussed and the proposed solutions are presented. The sensor node platform is built on Raspberry Pi 3 with ADMP401 microphones and sound card which can operate at a peak sampling rate of $48\ kHz$. Power consumption is small enough so that a $16\ Ah$ battery pack can support the nodes for more than $24\ h$ of measuring and data communication (without sleep mode).

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Received 09.11.2017 **Reviewer:** Associate Prof. PhD Vasil Galabov

Годишник на Технически Университет - София, том 67, книга 3, 2017
Proceedings of the Technical University of Sofia, Volume 67, Issue 3, 2017

МОБИЛНА СИСТЕМА ЗА МОНИТОРИНГ НА ШУМ - ПРОГРАМНО ОСИГУРЯВАНЕ И ЕКСПЕРИМЕНТАЛНИ РЕЗУЛТАТИ

Марин Б. Маринов

Резюме: *Настоящата статия е продължение на* [1]. *Тук също се разглежда процесът на проектиране и създаване на прототип на мобилна безжична система за измерване нивата на шум в околната среда. Докато в* [1] *този процес е обсъден от гледна точка на хардуера, тук вниманието е върху програмното осигуряване на системата, комуникацията и експериментални резултати. Системата е базирана на платформата Raspberry Pi 3 и икономически ефективни електронни компоненти.*

Контролни думи: *шум в околната среда, измерване на звуковото налягане, постоянното еквивалентно ниво на звуково налягане, Raspberry Pi.*

MOBILE NOISE MONITORING SYSTEM - SOFTWARE DESIGN AND EXPERIMENTAL RESULTS

Marin B. Marinov

Abstract: *In this paper the process described in* [1] *is discussed, namely the design and rapid prototyping of a wireless system for measuring environmental noise levels. Here the software design and the experimental results are considered. The sensing function, hardware design and energy consumption issues can be found in* [1]. *The system is based on the Raspberry Pi 3 platform and cost-effective electronic components.*

Key words*: environmental noise, sound pressure measurement, equivalent continuous sound pressure level, Raspberry Pi.*

1. SOFTWARE DESIGN

The architecture of the designed system follows the client-server communication model. The components and the connections between them are shown in Fig. 1. The server provides the user interface and the necessary resources for receiving and processing client requests. It is responsible for storing the information and retrieving it from the database.

Noise level measurements are available to end users via internet terminals. Any mobile or stationary device with a built-in browser such as a tablet, smartphone, or desktop computer can be used as a terminal to access the web-based interface.

ISSN 1311-0829

The system also has the capability for remote control and maintenance from a computer on which special management software is installed.

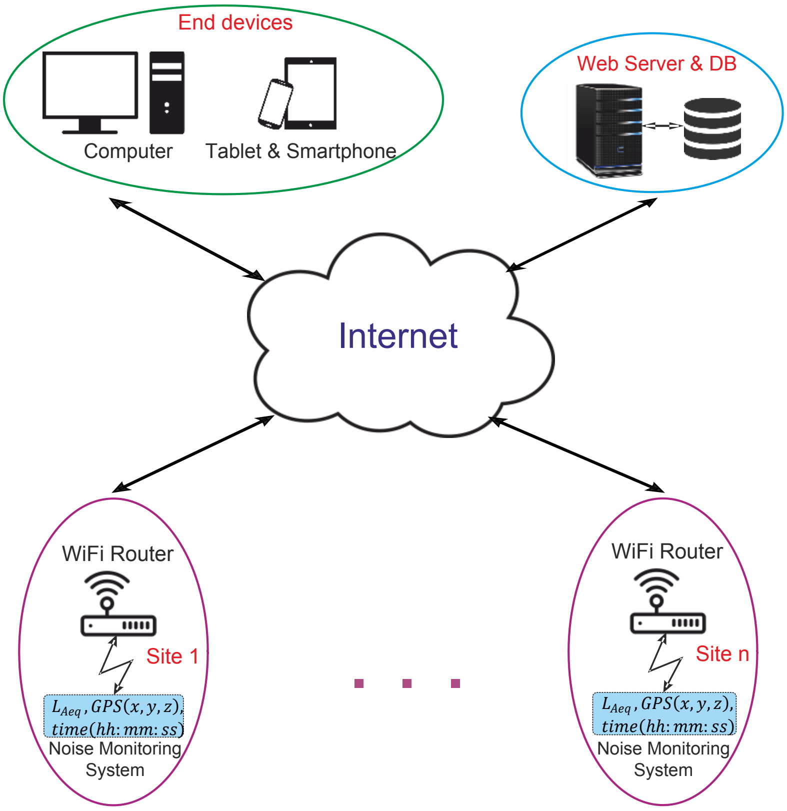

Fig. 1. Noise Monitoring System Architecture.

The communication between the elements in the structure is done through standard internet protocols. Their widespread use facilitates the integration of the system and the inclusion of additional measurement points in order to build a network of stations and expand the coverage area where noise levels are measured.

1.1 Software implementation

Software is written and tested on a Linux-based operating system. Specifically developed for Raspberry Pi is the Raspbian operating system distribution, which offers a graphical user interface (GUI) and use of the full potential of the available hardware resources [2].

The server part is implemented by the Apache Software Foundation software. Multiple APACHE-compliant modules allow the use of various encryption and content-limiting scripts. In the present study, the language used on the server is PHP. This is an open source language designed for web development, server application development, and dynamic web content. The code written in PHP is interpreted by the web server and the result returns to the web browser. In this way, the raw PHP code is not accessible for the user and ensures a higher level of security.

Remote control and software update modifications are possible with a VNC Viewer installed.

1.2. Algorithm of the main program

The main program algorithm is shown in Fig. 2. First, the system is initialized, which includes the configuration of the sound card and connection to the database. The program then enters an endless cycle of reading and processing incoming reports. The data is A-frequency and time weighted.

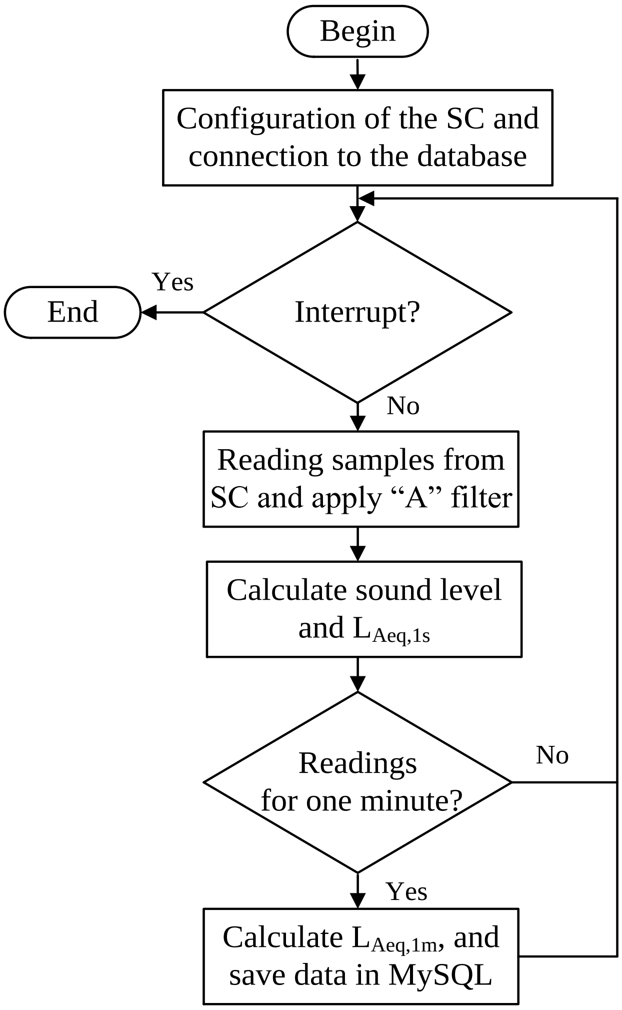

Fig. 2. Algorithm of the main program [3].

The A-filter weighting curve is defined as in ANSI S1.42.2001 [4] and attenuation limits as in IEC 61672-1:2002 [5]. The transfer function of the frequency filter is:

$$H_A(s) = G_A \frac{\omega_4^2 s^4}{(s+\omega_1)^2(s+\omega_2)(s+\omega_3)(s+\omega_4)^2}, \quad (1)$$

where $\omega_n = 2\pi f_n$ is the corner frequency and $f_1 = 20.598997\,Hz$, $f_2 = 107.70015 Hz$ $f_3 = 737.86223\,Hz$, $f_4 = 12194.217\,Hz$, G_A is a normalization coefficient used to normalize the function to unit amplification $(0\ dB)$ at 1 kHz.

In measurement systems together with frequency weighing, standard weighing (IEC 61672-1) is also applied [5]. It determines the "speed" with which the instrument responds to the change in noise levels. Most measuring devices use two conventional time weights "Fast" (F, Fast) and "Slow" (S, Slow). The "Fast" time weighing corresponds to $125\,ms$. It is usually used to measure noise which can vary widely over time. With "Slow" weighing sound levels are recorded at $1\,s$ intervals. This gives a better idea of average noise levels in an environment where it constantly changes.

The time weighting factor used is "S" or "Slow", which equals 1 second.

From weighted data, the sound pressure is calculated using the microphone sensitivity, the preamplifier gain factor and the sound card transmission coefficient:

The sensitivity at the output of the ADMP401 is given by the manufacturer and corresponds to $-42dBV$. The following correlations are used to obtain it in mV/Pa:

$$\text{Sensitivity dBV} = 20 \cdot \log_{10}\left(\frac{\text{Sensitivity mV/Pa}}{\text{Output A}_{\text{REF}}}\right), \tag{2}$$

where $\text{Output A}_{\text{REF}} = 1000\text{mV/Pa}$ is a reference value and Sensitivity -42dBV is the sensitivity of ADMP401.

From (2) it follows that:

$$\text{Sensitivity mV/Pa} = (\text{Output A}_{\text{REF}}) \cdot 10^{\frac{\text{Sensitivity dBV}}{20}}. \tag{3}$$

This results in Sensitivity of about $7.943 mV/Pa$.

In order to estimate the transfer coefficient of the sound card the experimental setup given in Fig. 3 is implemented. A 1 kHz sinusoidal signal with different amplitudes is generated from a DDS Function Generator (Instek SFG-1013 DDS). The signal amplitude is measured with True RMS digital multimeter (HP3478A DMM) and is stored through the sound card in memory in a waveform audio file format (wav).

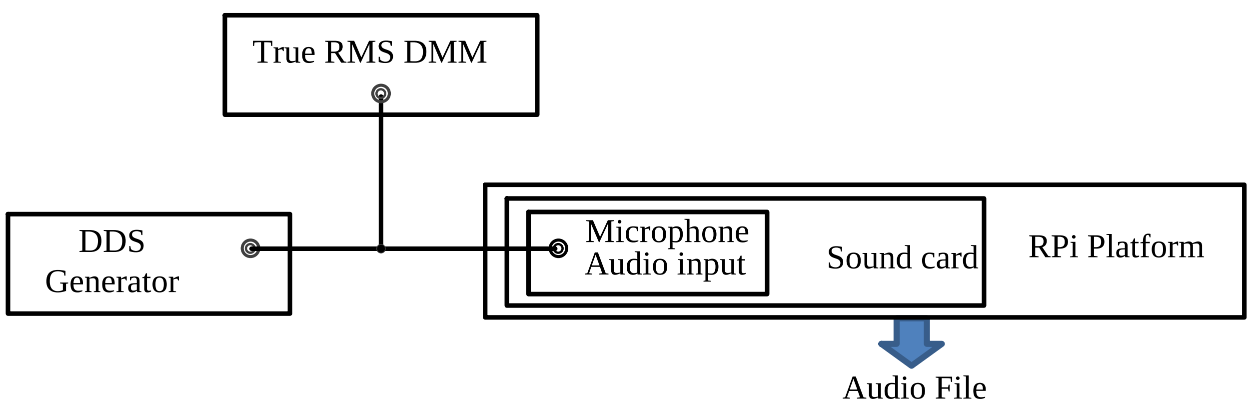

Fig. 3. The noise monitoring system.

The stored audio files with various amplitudes are processed in order to calculate their rms values. The transfer coefficient of the audio card can be obtained from the rms values read with DMM and the rms values calculated from the wav files.

After the transmission coefficients have been found, the relationship between the Pascal (Pa) input sound pressure and the resulting output digital value can be determined. This correlation is calculated as a coefficient in the program code. In the system functionality studies, coefficients approximately equal to $0.001214089\ Dig/Pa$ were obtained.

The algorithm continues with the calculation of L_{Aeq} values for one second period. In the database, these values are recorded as averaged over a 1 minute period, thus reducing the use of memory and processing time. For program interruption, a command is usually sent from the keyboard or so-called keyboard interrupt ($ctrl + c$) [6].

1.3. Algorithm for determination of the geographic position of the measurement system

A program for determining the exact geographic position of the measuring system starts working together with the main program. This is possible having the coordinates calculated by the GPS module. In the beginning, the program reads the date and time data from the built-in real-time clock and performs a setup of the system clock of the operating system (OS). The real-time clock is adjusted every time a GPS signal is available. Fig. 4 shows the generated geographic map. It is located on the home page of the user interface. The geographic coordinates of the system are displayed and a marker indicating the current location is added.

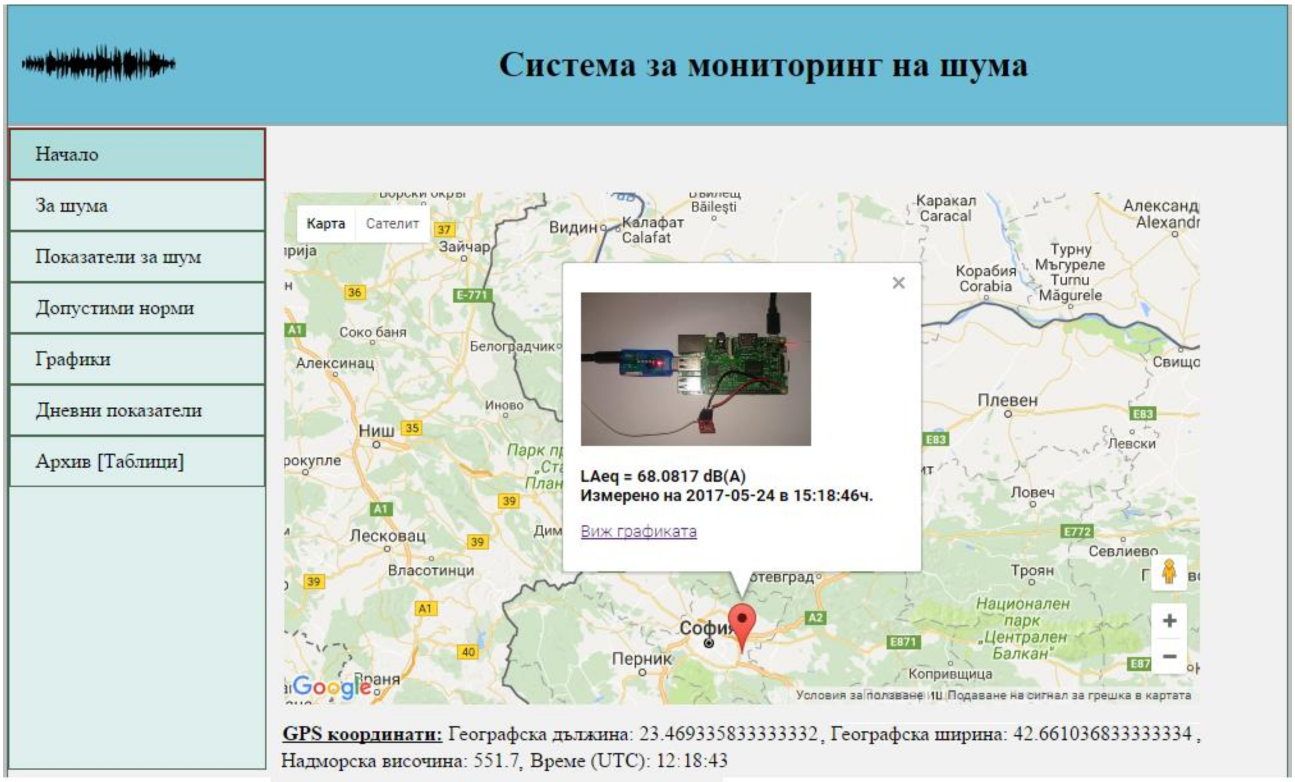

Fig. 4. Positioning of GPS coordinates on a geographic map.

The information from the GPS receiver is transmitted to the serial interface in the form of NMEA (National Marine Electronics Association) sentences. Each sentence consists of different data types and has its unique interpretation defined by the NMEA standard. For the needs of the project, one of the main GPS NMEA messages '$GPG-GA' was used. It contains all the information needed to determine the position using the geographic coordinates for latitude, longitude and altitude above sea level. Another sentence that is included in the shown algorithm and is used to set the date and time is '$GPRMC'

To display GPS coordinates in a graphical environment, the applied program interface API of Google Maps is used. It allows for the embedding of geographic maps in custom web pages using JavaScript. In this way, users can monitor the geolocation of the system in real time.

2. EXPERIMENTAL RESULTS

2.1 Calibration and accuracy test indoor

Type I integrating sound level meter Pulsar Model 91 [7] is used for calibration. The calibration procedure is conducted in a noise-proof room with the $1\ kHz$ sine wave calibration acoustic source. Acoustic signals with increasing amplitude in the interval of $[35, 100]\ dB$ and a duration of $10\ s$ are emitted towards both Type I reference sound level meter and the developed noise measurement system. The measured sound levels are saved for post-processing. The post-processing involves averaging the sound levels from the reference sound level meter and calculating the root-mean-square (A-weighted) of the sound card output. The A-filter weighting curve is defined as in ANSI S1.42.2001 [4].

Fig. 5 shows a comparison of 140 measurement samples from Pulsar 91 and the noise measurement system. The maximal difference between their values is about $\pm 1.8\ dB$.

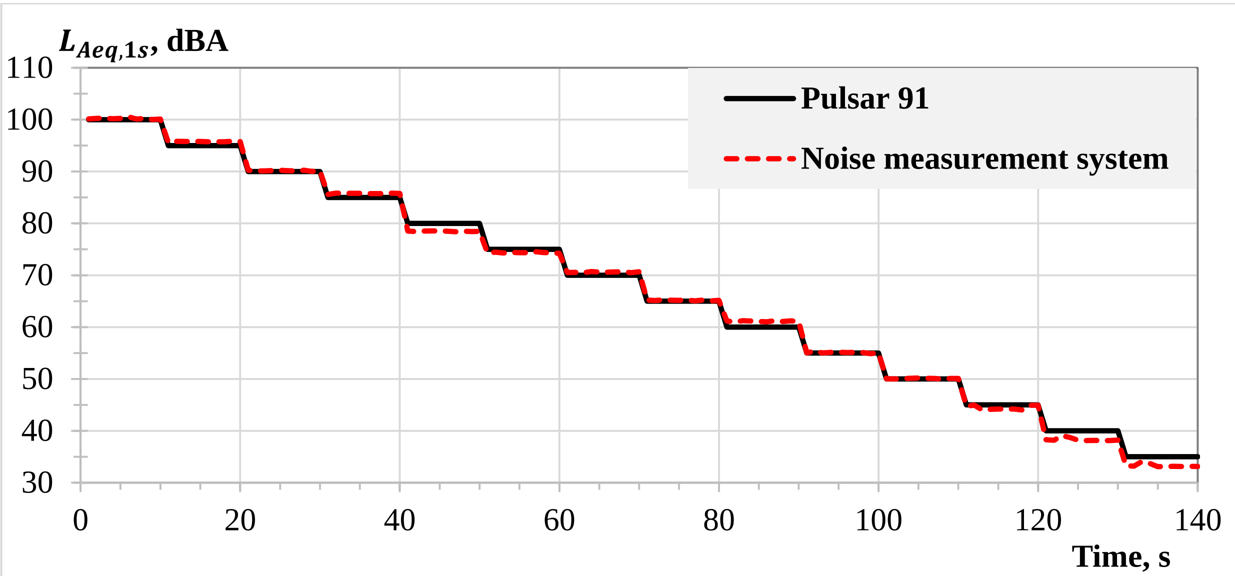

Fig. 5. Measured values of $L_{Aeq,1s}$ after calibration in the range [35,100] dB.

2.2. Accuracy test outdoor

A Pulsar Model 91 sound level meter was also used for the outdoor tests. The reference meter and the developed system were placed on a building façade at a height of $4\ m$ in a residential area in front of an intensive traffic road. Measurements of the sound pressure levels $L_{Aeq,1s}$ were made for duration of several minutes. The correlation with the reference instrument was very good. By different system outdoor tests correlation coefficients approximately equal to 0.95 were obtained. Results of the field test are shown in Fig. 6.

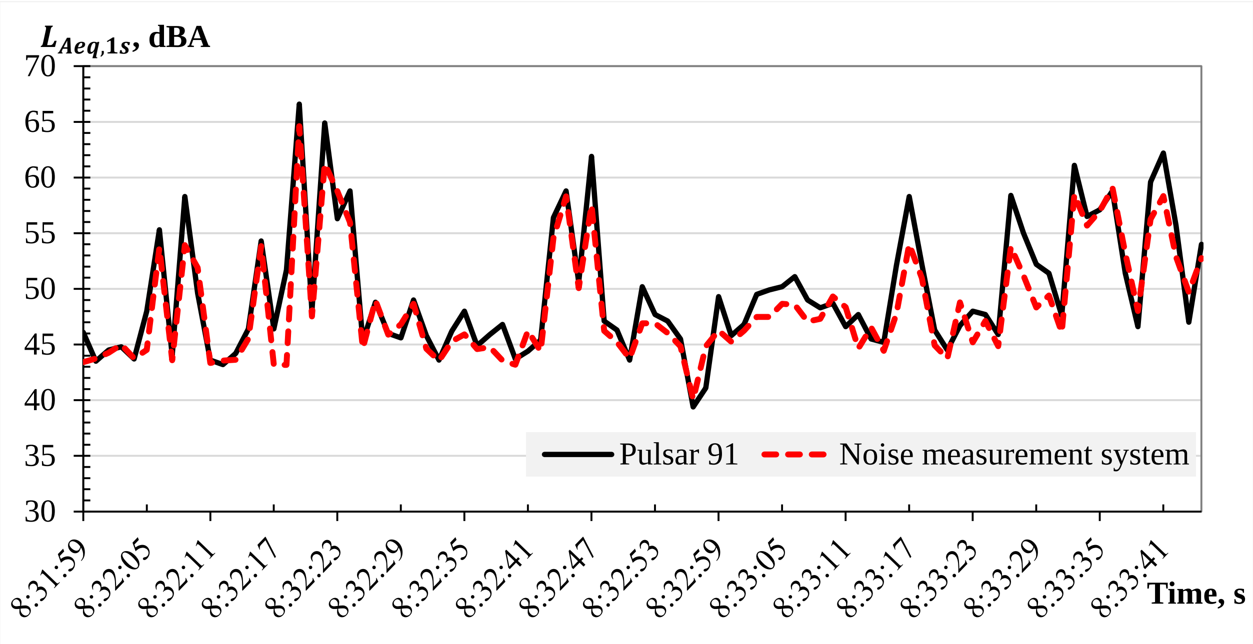

Fig. 6. Outdoor test results.

2.3 Database Structure and report generation

The information from the measurements is stored in a MySQL database. It is organized in a way that allows quick and easy content management and, in addition, the data security is achieved using unique keys and passwords. Access to MySQL is accomplished using specially developed PHP programs. PHP language makes possible the dynamic content management, reading, writing and performing of MySQL-specific actions.

For the purpose of the present project, the database is created and organized in the form of a table with three separate fields. In the first two fields the date and time are recorded in the following formats: *date (yy:mm:dd)* and *time (hh:mm:ss)* respectively. The third field serves to store the measured and A-weighted noise *equivalent sound pressure level* $L_{Aeq,1minn}$ of (float type data).

Records of the last 24 *hours* are always available to users and can be seen as a continuously updated chart of the internet platform. In addition, current values for the current day noise indicators are also given on the site.

The database information can also be used to generate advanced queries and reports for past periods. This is done by means of a code that runs on the server and the latter receives the input requests. A user-driven calendar is one of the ways for extraction of information. The integrated calendar uses CSS style description and JavaScript for dynamical update of the content. The PHP library JpGraph has been used for the image generation.

The software displays a graph of the instantaneous noise values for a desired calendar day. The database is referenced for a selected date or date and time. The values for the requested timeframe are transferred to a PHP-based code, which in turn draws the graph and displays it on the page (Fig. 7).

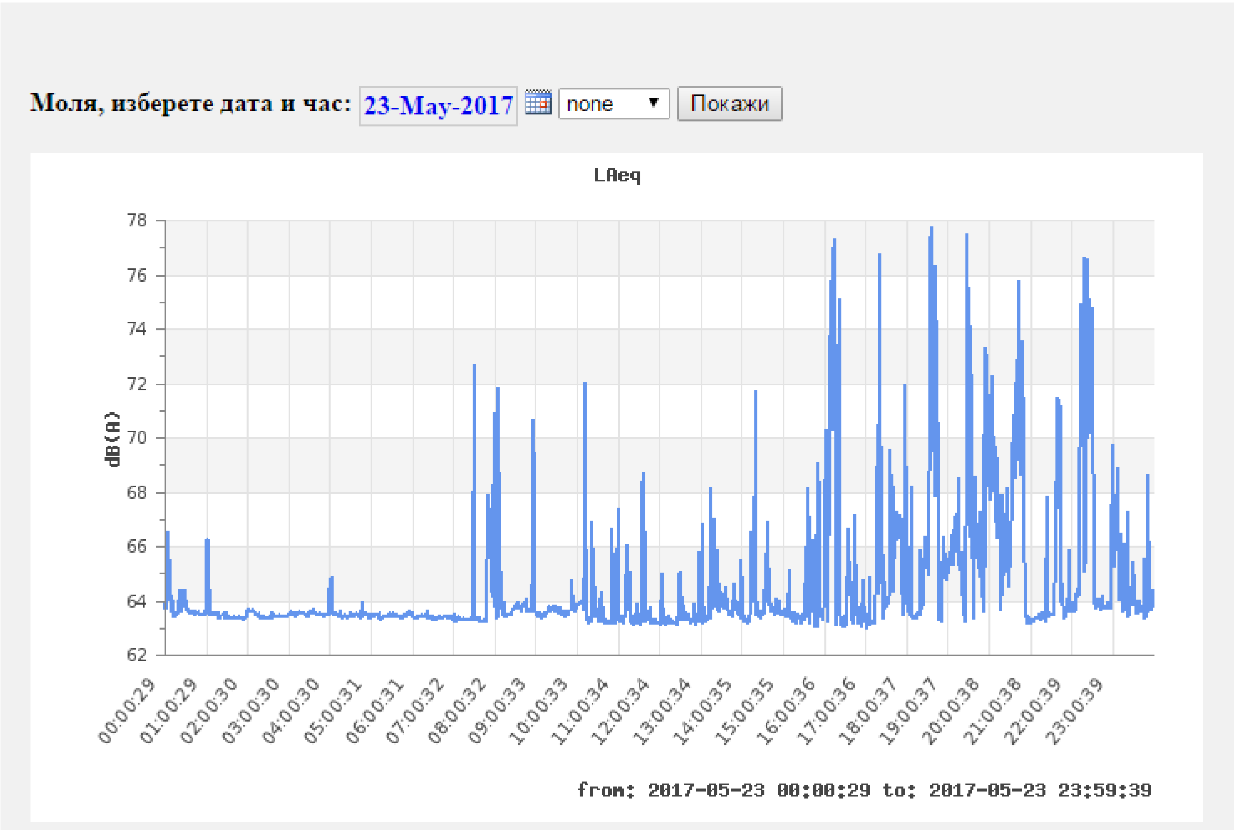

Fig. 7. Graph for noise levels on a given date (23.05.2017).

2.4. Daily noise parameters reports

The daily indicators $L_{day}, L_{evening}, L_{night}, L_{24}, L_{max}$ and L_{min} are graphically represented on a separate page of the web interface. Here there is also an option to set the number of previous days for which the parameters are to be displayed. Possible options are 3, 7, 14 or 21 days back. The selected period is three days, by default. The visualization of the results is done through bar plots [8].

The L_{24}, L_{max} and L_{min} readings are in a general bar chart, where the value of the corresponding parameter is displayed above each bar for clarity (Fig. 8).

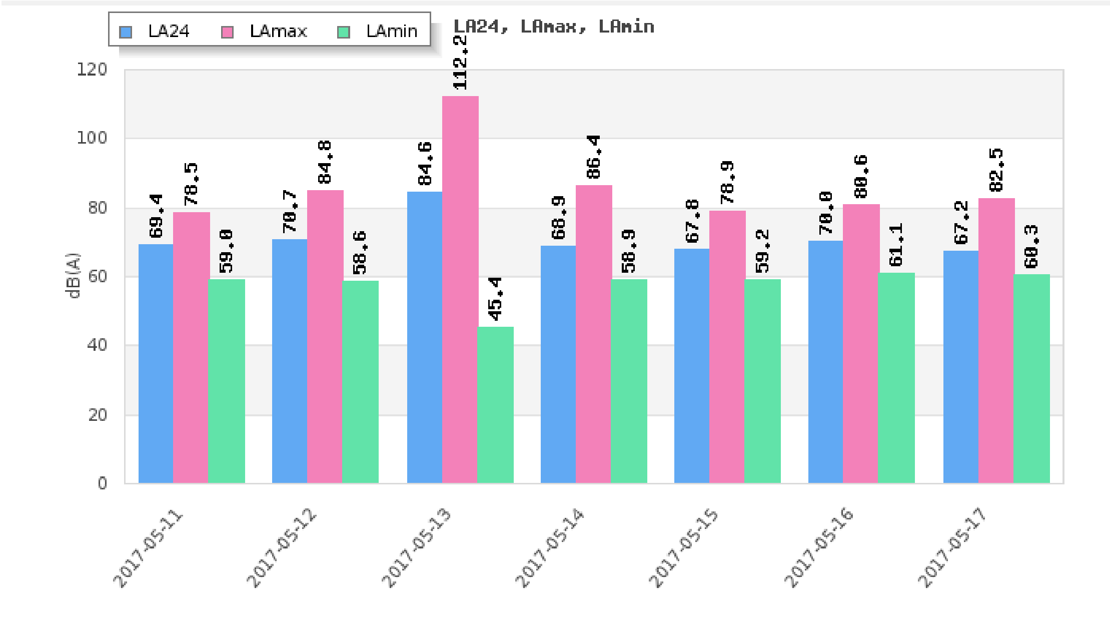

Fig. 8. Bar graph for L_{24}, L_{max} and L_{min}.

2.5 Noise parameters archiving

On a separate page there is an archive of the history of the pollution (Fig. 9). A request for a date lookup is formulated in order to review the values covering the period from the selected date over the previous 13 days. When no data is available for a particular day or parameter, the corresponding cells in the table are filled with dashes (-).

Дата	Lден, dB(A)	Lвечер, dB(A)	Lнощ, dB(A)	LA24, dB(A)	LAmax, dB(A)	LAmin, dB(A)
-	-	-	-	-	-	-
2017-05-23	65.76	68.28	63.66	71.2	77.77 измерено в 18:36:37ч.	62.99 измерено в 17:03:36ч.
2017-05-22	67.95	69.11	65.06	72.58	85.33 измерено в 16:21:25ч.	63.09 измерено в 00:18:18ч.
2017-05-21	69.03	68.37	63.29	71.68	78.74 измерено в 14:01:13ч.	59.99 измерено в 23:38:07ч.
2017-05-20	79.9	77.1	63.54	79.21	101.12 измерено в 14:58:03ч.	60.23 измерено в 08:55:00ч.
2017-05-19	64.28	63.72	62.66	69.41	79.31 измерено в 08:11:49ч.	61.25 измерено в 12:56:51ч.
2017-05-18	70.15	62.67	61.03	70.16	87.63 измерено в 18:19:43ч.	60.61 измерено в 00:04:35ч.

Fig. 9. Generation of a table for a chosen period.

5. CONCLUSION

The platform and the methods chosen to develop software for mobile noise monitoring system offer many advantages. The most important ones are the great versatility, the low price of the components and the easy integration of the final device in outdoors facilities, making it part of a sensor network as a node.

The acquisition of high-quality audio allows us to get more advanced results, like psychoacoustic parameters as future outcomes. Also the power of the Raspberry Pi as the core of the device makes it possible to perform the calculations on-board, instead of sending raw data to a sink node or to a server to do the calculations.

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Received 09.11.2017 **Reviewer:** Associate Prof. PhD Vasil Galabov

Годишник на Технически Университет - София, том 67, книга 3, 2017
Proceedings of the Technical University of Sofia, Volume 67, Issue 3, 2017

ИНТЕГРАЛНА РЕАЛИЗАЦИЯ НА СХЕМА ЗА ЗАЩИТА ОТ ПОВИШЕНА ТЕМПЕРАТУРА

Георги Савов, Георги Ангелов, Марин Христов

Резюме: *Разработена е схема за защита от повишена температура. Схемата съдържа подсхеми като band-gap ядро, аналогов двустъпален компаратор, тригер на Шмит, хистерезисна схема и резисторно разклонение за обратна връзка. Две температурно зависими напрежения, генерирани от band-gap ядро – едно с положителен и едно с отрицателен температурни коефициенти – са сравнени в аналоговия компаратор. Със стойността на това напрежение се задава горният температурен праг. Направена е също и хистерезисна схема за задаване долния температурен праг и предотвратяване на схемата от термална осцилация. Цялата схема е проектирана с дизайн кит на 0.18 μm технология на TSMC. Симулациите са извършени само на схемно ниво.*
Ключови думи: *схема за защита от повишена температура, горен температурен праг, долен температурен праг, независим от кристала на чипа.*

IC IMPLEMENTATION OF AN OVER TEMPERATURE PROTECTION CIRCUIT

Georgi Savov Jr., George Angelov, Marin Hristov

Abstract: *Over temperature protection circuit has been developed. The circuit contains several sub-circuits such as band-gap core, analog two-stage open loop comparator, Schmitt trigger, hysteresis circuit, and feedback resistor branch. Two temperature dependent voltages generated from band-gap core – one with positive and one with negative temperature coefficients – are compared in analog comparator. By controlling the value of that voltage with positive temperature coefficient, the upper threshold temperature is precisely set. A hysteresis circuit, which is used to set the lower threshold temperature and to prevent the circuit from thermal oscillation, is also made. This overall circuit is designed in TSMC 0.18 μm technology kit. The simulations are performed on schematic level only.*
Keywords: *over temperature protection circuit, upper threshold temperature, lower threshold temperature, independent from the die.*

INTRODUCTION

Over temperature protection circuits are often required in some integrated circuits and devices, which are working under risk of getting self-heated, or devices which are exposed to risk of getting the values of surrounding temperature above the critical

ISSN 1311-0829

values specified in chip specifications [1].

The main requirement for such circuit is that it should not disturb normal operation of the die in normal working conditions. This means that threshold temperatures for turning on and off the protection circuit should be precisely set and also independent from process and mismatch variations. Power consumption and size of the circuit are big concern as well.

There are several ways to realize over temperature protection circuit. One simple circuit can be made by comparing two voltages – one from p-n junction with negative temperature coefficient, which is CTAT (Complementary to Absolute Temperature) voltage, and one reference voltage set by resistor and current mirror. The main disadvantage in such structure is the big process dependence of the value of the resistor, so the reference voltage will vary with process. This will make the threshold temperature process dependent, so in some corner conditions the protection circuit will not protect the die from over temperature [1].

CIRCUIT DESIGN

In this paper we propose an over temperature protection circuit, which is almost process insensitive. This circuit provides precise control of the upper and lower threshold temperatures by a hysteresis circuit. This circuit is also almost independent from the die, in which it is going to be implemented, because all the voltages needed for the proper operation of the protection circuit, are generated in the band-gap core; excepted are the supply signals and the enable disable signals, which should be generated from external digital logic.

Fig. 1 shows the main control circuit of the proposed over temperature protection circuit. It consists of an analog two-stage open-loop comparator, Schmitt trigger, hysteresis circuit, and a feedback resistor branch. It should be noted that there are couple of transistors in the circuit, which are used to enable or disable the protection circuit.

The analog comparator in Figure 1 compares two voltages with positive and negative temperature dependent voltage coefficients, which are generated from the band-gap core. The positive dependent voltage is applied to the positive input of the comparator, while the negative dependent voltage is applied to the negative input of the comparator.

The output voltage of the comparator is buffered by a couple of invertors and a Schmitt trigger and this buffered signal is the output of the protection circuit. It should be noted that an inverse voltage (inversed to the output voltage) is generated in the Schmitt trigger; this voltage is used in the hysteresis circuit.

In Fig. 1 the hysteresis circuit is composed of nMOS transistor M9 and resistor R20. The working principle is simple – voltage is applied to the gate of M9, which is inversed to the output of the over temperature protection circuit, so that when the protection circuit is off, M9 is on, and it is providing path between "HYST" node and ground. When the protection circuit turns on, M9 turns off, and the voltage drop across R20 resistor is added to the voltage on "iptat" node, which has positive temperature coefficient. By adding hysteresis in the circuit thermal oscillation independence is achieved as well as a precise control over the lower threshold temperature.

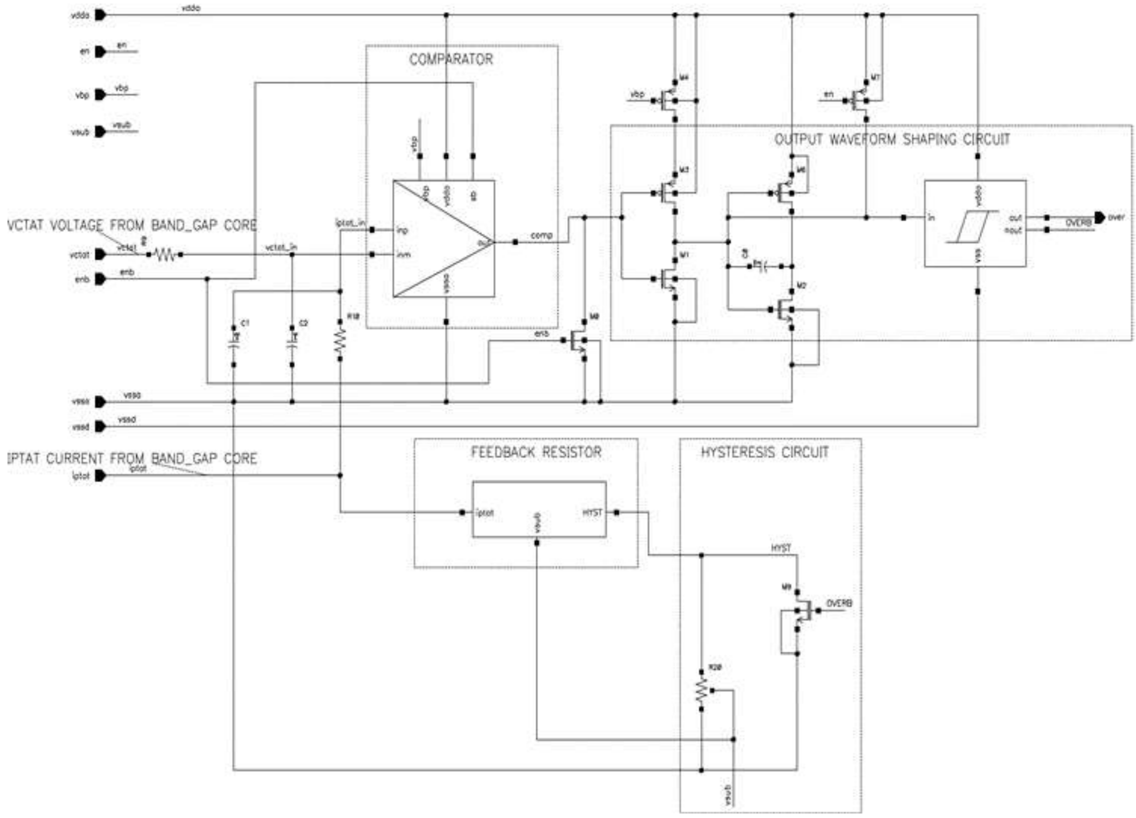


Fig. 1. Main control circuit.

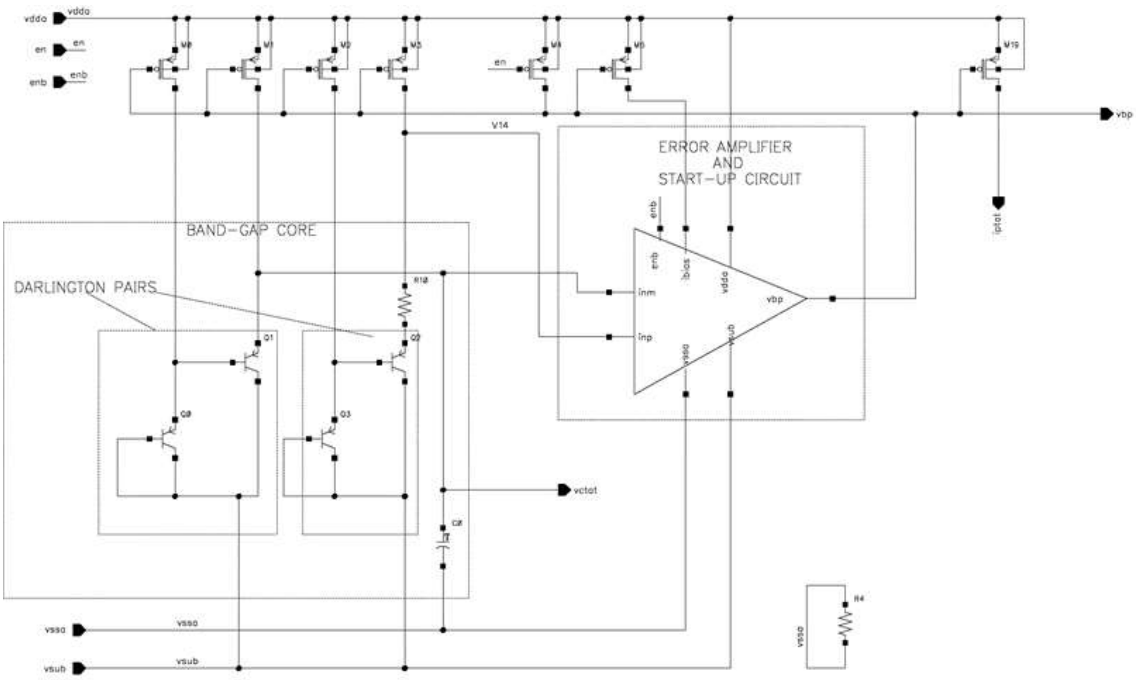


Fig. 2. Band-gap core used in this paper.

In Fig. 1 the feedback resistor is made of series connected resistors, which are used to set the voltage on "iptat" node to precise value. By doing this the upper threshold temperature is precisely controlled.

The RC filters on the inputs of the analog comparator in Fig. 1 are set with different values. The capacitor on the positive input of the comparator is set much bigger than the capacitor on the negative input. This is done so, because in start-up condition, when the supply is ramping from 0 to vdda, the voltage on the positive input of the comparator could ramp faster than that on the negative input and this could lead to wrong switching to high state on the output of the protection circuit.

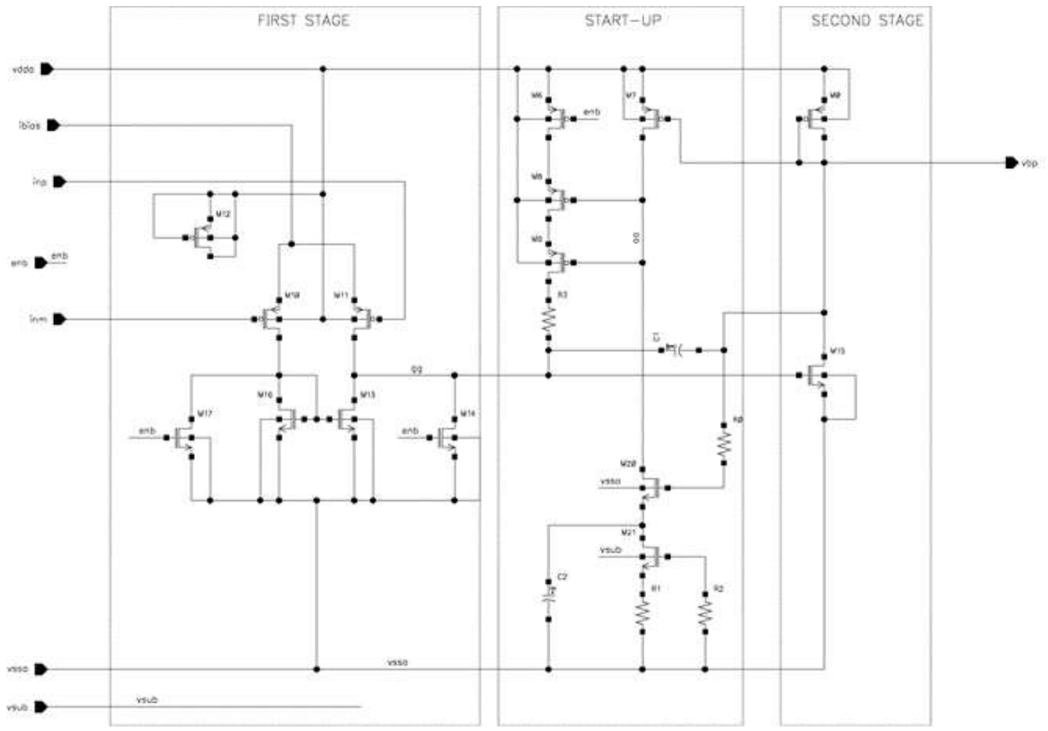
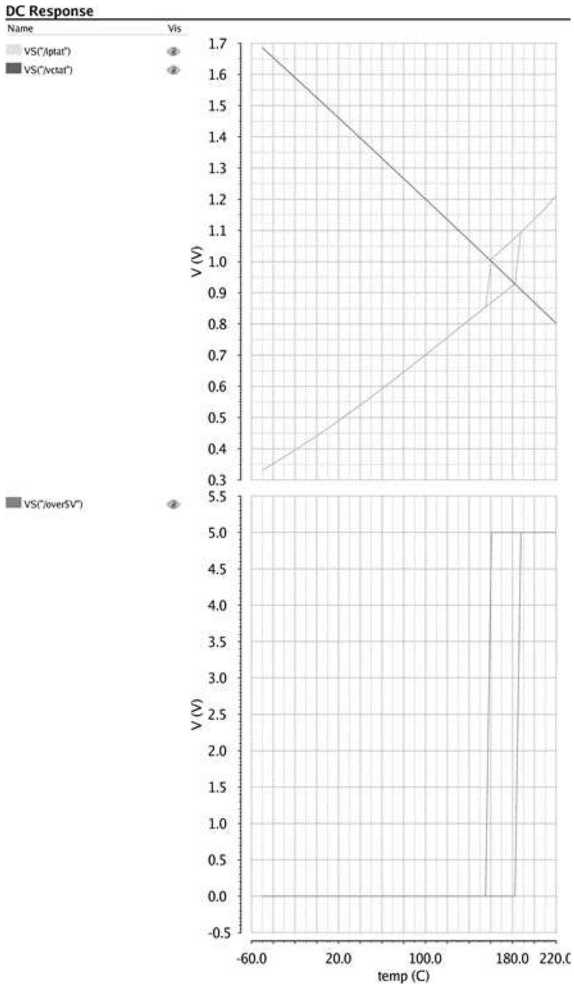


Fig 3. Error amplifier with start-up circuit.

Fig. 2 shows the proposed band-gap core used in this paper [2]. The main point here is that it is made of two Darlington pairs instead of two p-n junctions of diode connected bipolar transistors, because the Darlington pair has about 2 times bigger temperature dependence from that of one p-n junction. The voltage with negative temperature coefficient (CTAT) is generated from the first Darlington pair.

The voltage with positive temperature coefficient (PTAT – Proportional to absolute temperature) is generated from the voltage drop over the resistor R10 (in which the temperature dependence is known by "φT" – the thermal potential so that the current mirrored on the "iptat" pin is also with positive temperature coefficient. It should be noted, that the start-up circuit of the band-gap core is designed in the error amplifier.

In addition, the bias voltage "vbp" is generated on the output of the error amplifier in Fig. 3, where it is set by the voltage drop over diode connected M0 transistor.



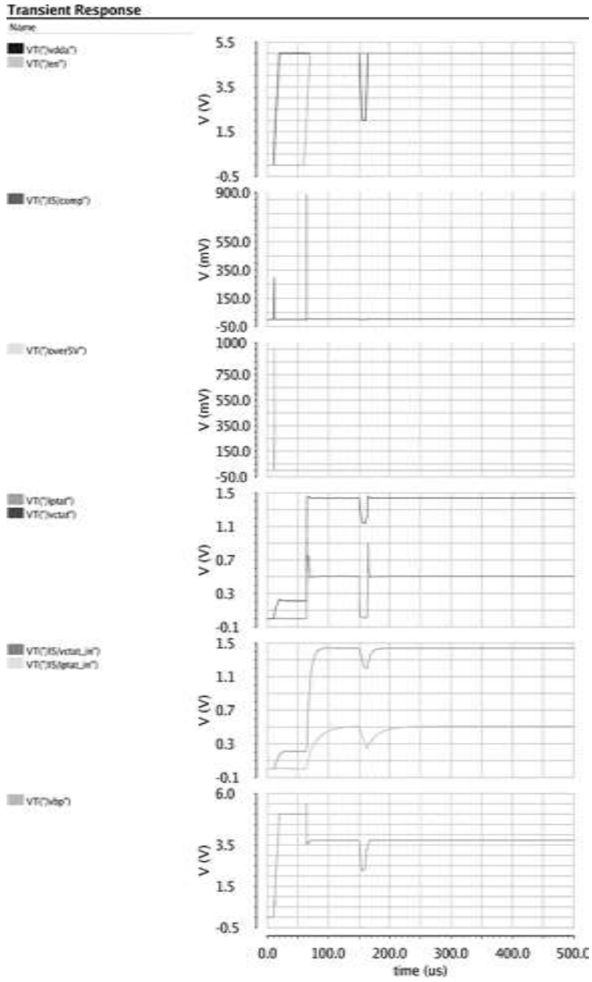


Fig. 4. Result of the DC analysis.

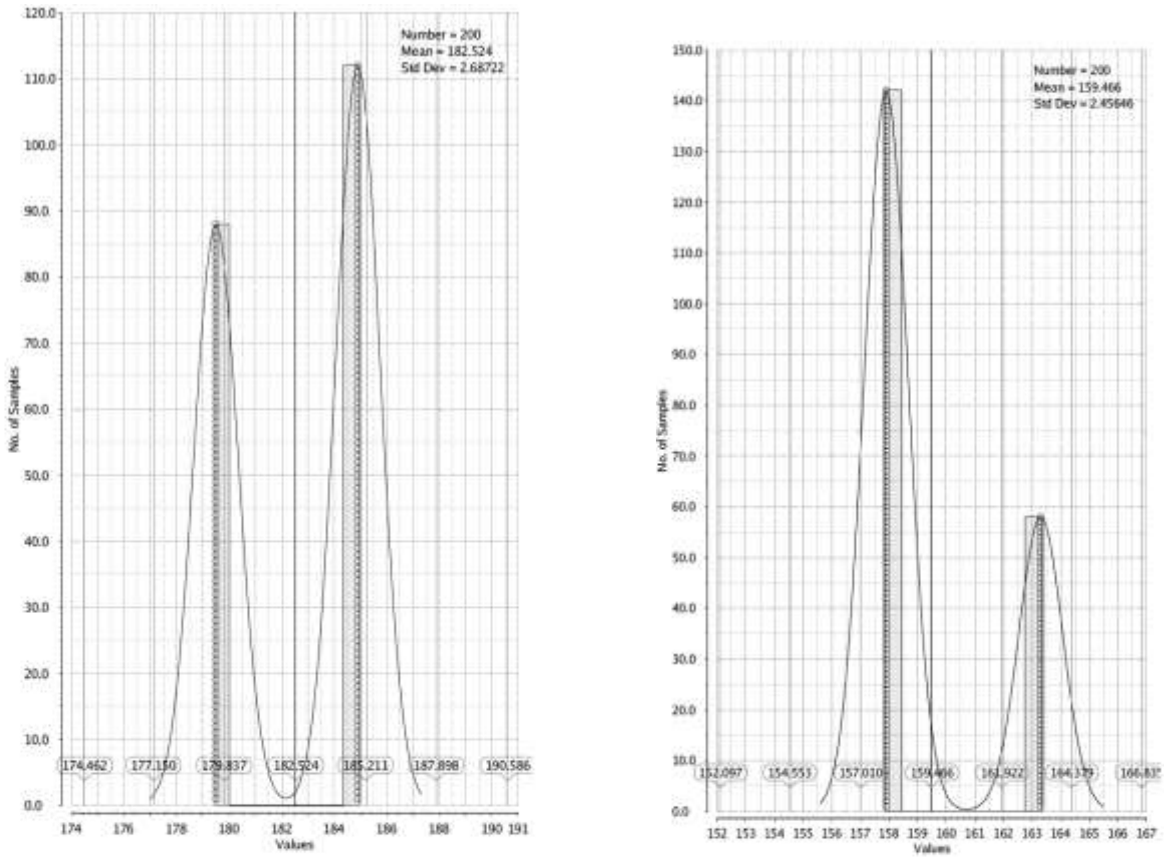
Fig. 5. Result of the transient analysis in start-up of supply voltage with 10 µs ramp and simulated voltage drop in the supply voltage at 150 µs simulation time.

SIMULATION AND EXPERIMENTAL RESULTS

Fig. 4 shows the result from DC hysteresis simulation sweeping the temperature. In this figure the working principle of the protection circuit could be explained. When the voltage on the “iptat” node becomes positive enough, or in other words the temperature reaches the threshold temperature which is set and reaches the voltage value of the “vctat” node, the output of the block changes from low to high state and this signal should work like a disable for the die, in which the protection circuit is implemented. This threshold temperature is set by setting the voltage on the “iptat” node, using the Feedback resistor, consisting of series of connected resistors with total resistance of the hole sub-circuit about 37.5 kΩ. When temperature starts to drop, the two voltages on “vctat” and “iptat” nodes cross each other at precisely defined threshold and the output of the block triggers back to low state. This voltage level is set by putting the voltage drop across R20 to an exact value. In our design the upper threshold temperature is set to 182 ºC and the lower threshold temperature is set to 160 ºC.

Fig. 5 shows the result of the transient simulation. It starts up when the supply voltage ramps from 0 to vdda and also voltage drop is simulated in the supply voltage at 150 μs simulation time. VT/I5/"iptat_in" and VT/I5/"vctat_in" are the voltages on the two inputs of the analog comparator and as we can see VT/I5/"iptat_in" voltage starts slower, because of the reason mentioned before.

Fig. 6 shows the results of the Monte Carlo analysis on schematic level for the upper – 6-a) and lower – 6-b) threshold temperatures. It can be seen that the upper threshold temperature varies with 5 ºC and the lower threshold temperature varies with 6 ºC from their set values in worst-case process and mismatch variations.



a) **b)**

Fig 6. Monte Carlo analysis for the a) upper & b) lower threshold temperatures.

CONCLUSION

In this paper an over temperature protection circuit with precisely controlled upper and lower threshold temperatures is introduced. All the devices used in the proposed circuit are 5 V devices within TSMC 0.18 μm technology. At schematic level the circuit shows promising simulation results, which prove it to be nearly process insensitive.

The developed protection structure is independent from other electrical blocks in the die, in which it is implemented because all the voltages needed for the circuit to work properly are generated in the designed band-gap core, except the digital enable signals and the supply voltages.

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Received: November 10, 2017 **Reviewer**: Prof. PhD Slavka Tzanova

Годишник на Технически Университет - София, том 67, книга 3, 2017
Proceedings of the Technical University of Sofia, Volume 67, Issue 3, 2017

ПРОЕКТИРАНЕ НА ПОЛИМЕРНА ЧЕРВЯЧНА ПРЕДАВКА ЗА ЛИНЕЙНО УПРАВЛЕНИЕ НА РОБОТ С ПАРАЛЕЛНА КИНЕМАТИКА

Кирил Николов

Резюме: *Настоящият доклад се концентрира върху разработен индустриален робот с паралелна кинематика и две степени на свобода, видовете термопласти за производство на пластмасови зъбни колела, полимерни материали за създаване на композитни зъбни колела, софтуерната среда на програмния пакет Autodesk® Inventor®, както и върху метода за проектиране на червячна зъбна предавка за линейно управление на индустриалния робот с паралелна кинематика и две степени на свобода и крайния резултат от проектирането.*
Ключови думи: *индустриален робот с паралелна кинематика и две степени на свобода, термопласти за пластмасови зъбни колела, полимерни материали композитни зъбни колела, Autodesk® Inventor®, червячна зъбна предавка*

POLYMER WORM GEAR DESIGN FOR LINEAR CONTROL OF A ROBOT WITH PARALLEL KINEMATICS

Kiril Nikolov

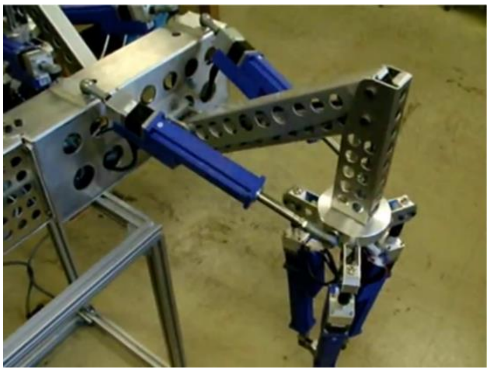
Abstract: *This report focuses on a developed industrial robot with parallel kinematics and two degrees of freedom, the types of thermoplastics for manufacturing plastic gears, polymer materials for creating composite gears, the software environment of the Autodesk® Inventor® software suite, the worm gearing design for linear control of the industrial robot with parallel kinematics and two degrees of freedom and the end result of the design procedures.*
Key words: *industrial robot with parallel kinematics and two degrees of freedom, thermoplastics for plastic gears, polymer materials for composite gears, Autodesk® Inventor®, worm gear, industrial robot with parallel kinematics and two degrees of freedom*

1. ВЪВЕДЕНИЕ

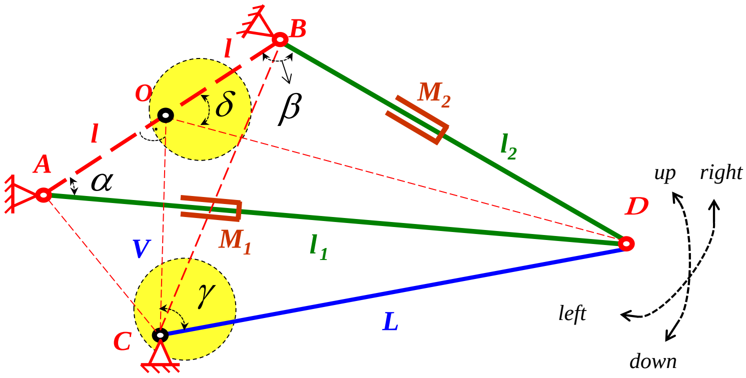
Целта на разработката е проектирането на задвижващите звена на прототип на индустриален робот (фиг.1) с паралелна кинематика и две степени на свобода [4]. Задачата, поставена в нейното изпълнение е проектиране на полимерна червячна предавка за линейно управление на робота с използване на програмния пакет *Autodesk® Inventor®* и модула му за проектиране на зъбни колела *Power Transmission*.

ISSN 1311-0829

Прототипът на индустриалния робот (фиг.1) с паралелна кинематика е представен в [5] и с: • схема на механиката (фиг.2); • с размерите на проектираната конструкция (табл.1); • аналитично описание на решенията на задачата за права и обратна кинематика (табл.2); • диаграма (фиг.3) на работното пространство за движение на точка ***D*** (фиг.2); • ***3D***- сферични изображения на зависимостите (1), (2); • ***3D***-контурно-параметрични изображения на $\gamma\,(l_1, l_2, n)$ и на $\delta\,(l_1, l_2, q)$, показани на фиг.4 ÷ фиг.7.



Фиг.1. Прототип на робота.



Фиг.2. Схема на механиката на робота.

Таблица 1.

Размери на проектираната конструкция

1.	$AO = OB = l = 150\,[mm] = 0.15\,[m]$	**4.**	$M_1, M_2 - travel = 200\,[mm] = 0.2\,[m]$
2.	$OC = v = 150\,[mm] = 0.15\,[m]$	**5.**	$\Delta l_1 = \Delta l_2 = \pm 100\,[mm] = \pm 0.1\,[m]$
3.	$CD = L = 1000\,[mm] = 1\,[m]$	**6.**	$l_1\,max - l_1\,min = 200\,[mm] = 0.2\,[m]$

Таблица 2.

Решения на задачата за права и обратна кинематика

Права кинематика за вертикално движение на точка ***D***

$$\gamma(l_1, l_2) = 2arctg\sqrt{\frac{(l_1^2 + l_2^2) - m}{n - (l_1^2 + l_2^2)}}, \left(m = 2\left(l^2 + (L - v)^2\right); n = 2\left((v + L)^2 + l^2\right)\right) \quad (1)$$

Права кинематика за хоризонтално движение на точка ***D***

$$\delta(l_1, l_2) = arccos\left(\frac{l_1^2 - l_2^2}{\sqrt{ql_1^2 + ql_2^2 - 1}}\right), \left(q = \frac{1}{2l^2}\right) \quad (2)$$

Обратна кинематика за вертикално движение на точка ***D***

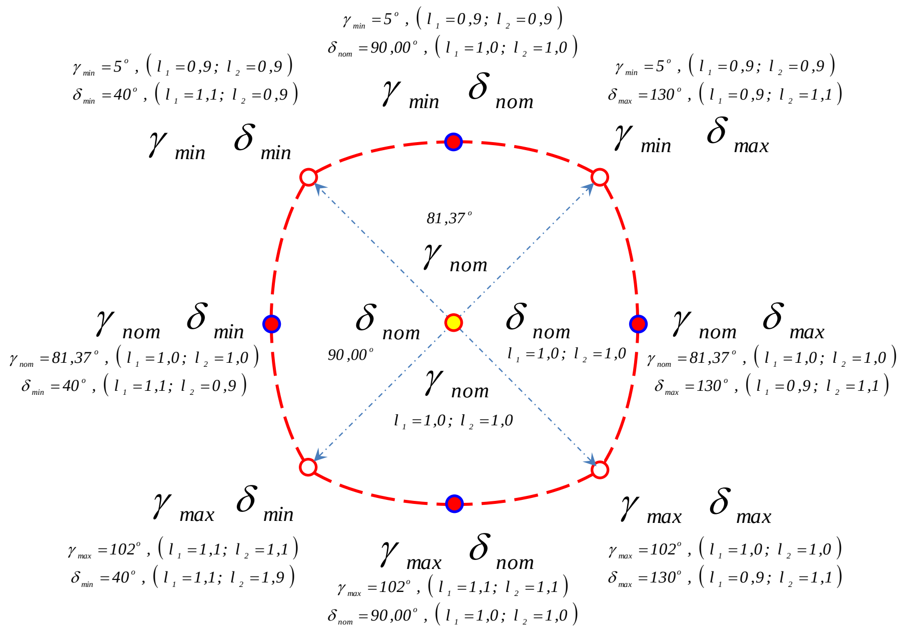
$$l_1(\gamma) = \sqrt{\left(m + n\left(tg\frac{\gamma}{2}\right)^2\right)\left(1 + \left(tg\frac{\gamma}{2}\right)^2\right)^{-1} - l_2^2}, \left(l_2 = const; m = const; n = const\right) \quad (3)$$

$$l_2(\gamma) = \sqrt{\left(m + n\left(tg\frac{\gamma}{2}\right)^2\right) \cdot \left(1 + \left(tg\frac{\gamma}{2}\right)^2\right)^{-1} - l_1^2}, \left(l_1 = const; m = const; n = const\right) \quad (4)$$

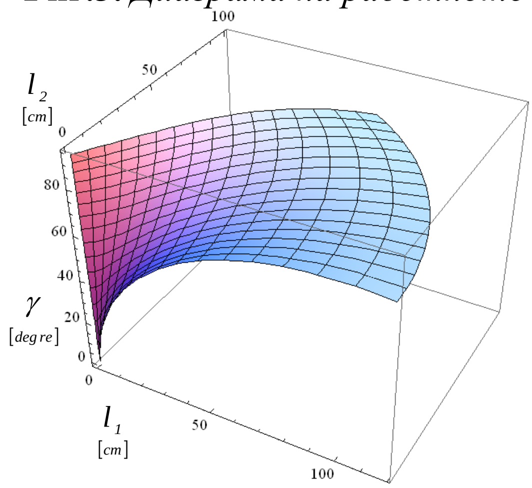
Обратна кинематика за хоризонтално движение на точка ***D***

$$l_1(\delta) = \sqrt{\frac{\left(2l_2^2 + q(cos\delta)^2\right) - \sqrt{\left(2l_2^2 + q(cos\delta)^2\right)^2 - 4\left(0.5l_2^{-2} - l_2^4 + q(cos\delta)^2\right)}}{2}}, \left(l_2 = const; q = const\right) \quad (5)$$

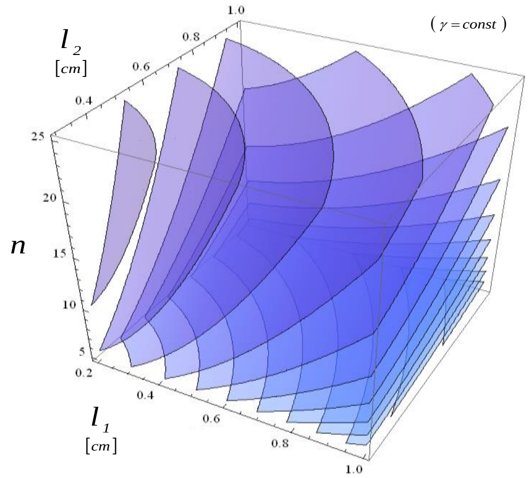
$$l_2(\delta) = \sqrt{\frac{\left(2l_1^2 + q(cos\delta)^2\right) + \sqrt{\left(2l_1^2 + q(cos\delta)^2\right)^2 - 4\left(-0.5l_1^{-2} - l_2^4 + q(cos\delta)^2\right)}}{2}}, \left(l_1 = const; q = const\right) \quad (6)$$



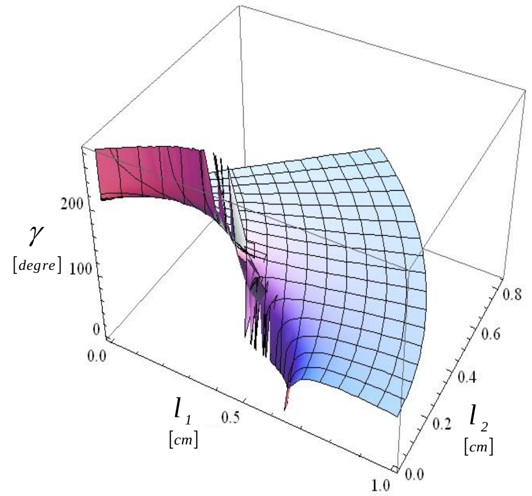
Фиг.3. *Диаграма на работното пространство за движение на точка* **D.**



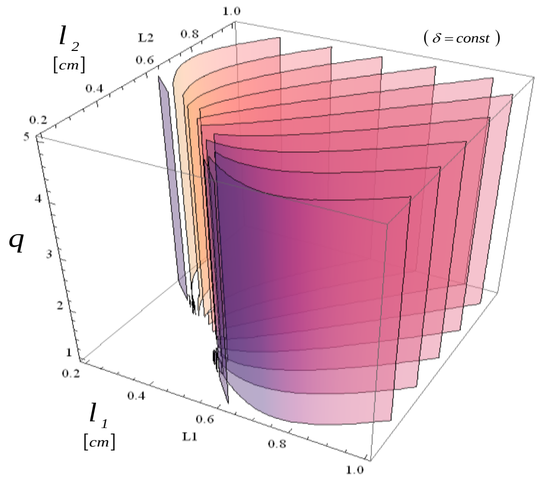
Фиг.4. ***3D****-мерно сферично изображение на зависимостта на ъгъла гама* γ *от дължината на рамената* l_1 *и* l_2.



Фиг.5. ***3D****-мерно контурно-параметрично изображение на зависимостта* $\gamma\ (l_1, l_2, n)$ *от стойностите на* l_1, l_2, n.



Фиг.6. ***3D****-мерно изображение на зависимостта на ъгъла* δ *от дължината на рамената* l_1 *и* l_2.



Фиг.7. ***3D****-мерно контурно-параметрично изображение на зависимостта* $\delta\ (l_1, l_2, q)$ *от стойностите на* l_1, l_2, q.

2. ТЕРМОПЛАСТИ ЗА ПЛАСТМАСОВИ ЗЪБНИ КОЛЕЛА [1]

Избирането на термопласти за изработване на пластмасови зъбни колела зависи от редица фактори: • приложимост за леене под налягане; • възможност за изработване в сравнително тесни граници на допусковите полета; • постоянство на размерите в производствени условия (влажност, топлина и др.); • запазване на формата при работа (коравина); • малък коефициент на триене; • възможно по-голям Е-модул, осигуряващ необходимата твърдост и достатъчна ударна жилавост; • устойчивост на мажещи средства и температурни влияния; • икономическа ефективност от използването им.

За изработване на зъбни колела най-подходящи са различните видове *полиамиди, полиоксиметилени* (*полиформалдехиди*) и *полиестери*. Оценката е ориентировъчна поради неравнозначното влияние на отделните фактори. Повечето от термопластите се изработват в широка гама различно модифицирани типове. Отделни производители предлагат полиамиди и полиоксиметилени, предназначени специално за изработване на зъбни колела. Свойствата на избраната пластмаса трябва да отговарят на производствените условия, които включват сложен комплекс от изисквания. Ето защо всяко уточняване на вида на пластмасата за изработване на зъбни колела представлява компромис. От решаващо значение за определяне на подходяща пластмаса е видът на натоварването:

• при голяма периферна сила и малка периферна скорост са подходящи жилавите и по-твърди пластмаси; използват се ненапълнени и стъклонапълнени термопласти ПА 6, ПА 11, ПА 12, ПА 66 СН, ПЕТФ СН;

• при голяма периферна сила и скорост в предавката се получава загряване от повишеното триене между профилите на зъбите, от вътрешното триене и поради нарасналото динамично натоварване. В този случай от значение е топлинната формоустойчивост и добрата плъзгаща способност (малък коефициент на триене) при повишена температура на експлоатация; подходящи термопласти са ПОМ, ПЕТФ, ПБТФ, ПЕТФ СН;

• при малка периферна сила и голяма периферна скорост е възможно появяване на шум; подбира се материал с висока шумозаглушаваща способност като ПА 6, ПА 66, ПЕТФ, ПУР (фиг.8); - при натоварване на удар и често обръщане на посоката на въртене се подбират термопласти с по-голяма ударна жилавост и добри демпфиращи свойства - ПА 6, ПА 66, ПА 66 СН, ПА 6 СН, ПК, ПА 11, ПА 12, ПЕТФ, ПБТФ;

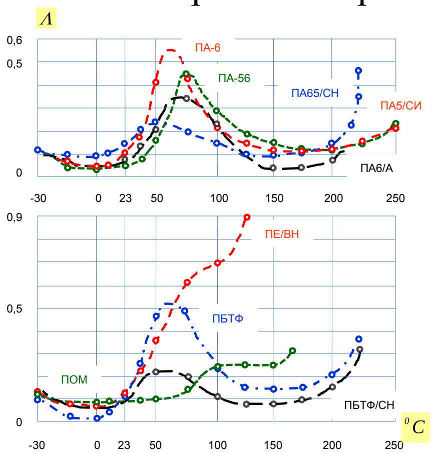
• при голям брой работна цикли (до разрушаване) се използват устойчиви на умора (фиг.9) с добра плъзгаща способност пластмаси, като ПА 66, ПОМ, ПЕТФ, които имат малък коефициент на триене и добра износоустойчивост.

За якостните промени в материала в зависимост от температурата се съди по стойностите на Е-модула (фиг.10 и фиг.11). От диаграмите се вижда положителното влияние на пълнежа от стъклени влакна върху твърдостта на пластмасата при повишена температура. Ударната жилавост (образци с надрез) на ПА 66 нараства с повишаване на влагосъдържанието. Описаните свойства дават основание за следните изводи:

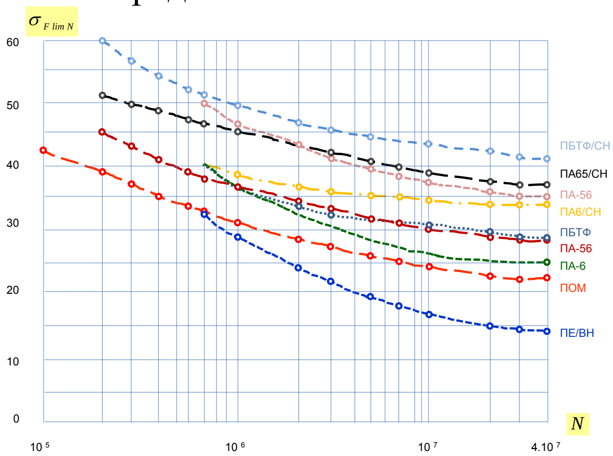
● в сравнение с металните зъбите на колела от полиамиди при натоварване деформират еластично повече, поради което за пластмасовите зъбни колела не са целесъобразни големи изисквания за точност;

● при еднакво натоварване деформирането на профила на зъба е толкова по-голямо, колкото по-малък е модулът на линейна деформация;

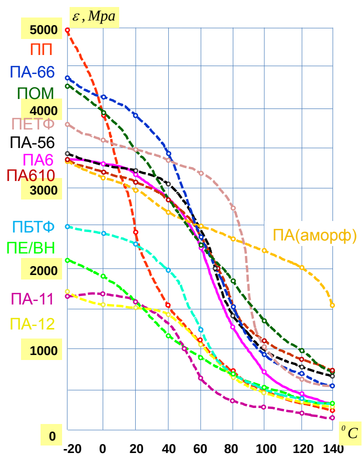
● тъй като зъбите на пластмасовите зъбни колела се деформират сравнително лесно, изпресовки по ръбовете на зъбите и малки неточности от монтажа не се отразяват неблагоприятно на работата на предавката.



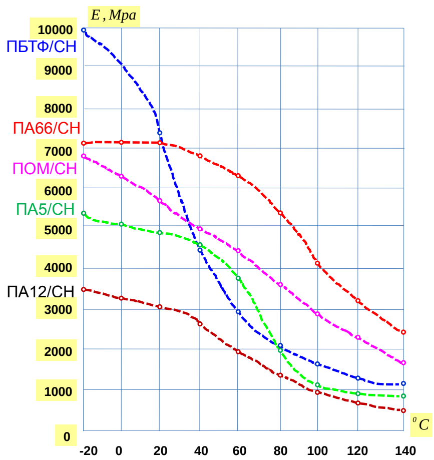
Фиг.8. *Логаритмичен декремент на виброгасене в зависимост от температурата T*. (*адаптирана от* [1])



Фиг.9. *Якост на променливо огъване (циклично огъване)* $\sigma_{F\lim N}$ *за различни пластмаси в зависимост от броя на циклите на натоварване.* (*адаптирана от* [1])



Фиг.10. *Стойности на E -модула в зависимост от температурата T за различни пластмаси.* (*адаптирана от* [1])



Фиг.11. *Стойности на T -модула в зависимост от температурата T за различни стъклонапълнени пластмаси.* (*адаптирана от* [1])

3. ПОЛИМЕРНИ МАТЕРИАЛИ ЗА ЗЪБНИ КОЛЕЛА

Полимерните материали, използвани в производството на зъбни колела са описани в [6]. Тук е представена обща информация за всеки един от материалите:

- **POM** – Полиоксиметилен (POM), известен още като ацетал, полиацетал и полиформалдехид, е инженерен термопласт използван в прецизни части, изискващи висока твърдост, нисък коефициент на триене и отлична стабилност на размерите. Типичните приложения за инжекционно формован POM включват високоефективни инженерни компоненти като малки зъбни колела, рамки за очила, сачмени лагери, ски-вериги, крепежни елементи, пистолети, дръжки за ножове и заключващи системи.

- **PA6** – Полиамид 6 (PA6), найлон 6 или поликапролактам е полимер, разработен от Пол Шлак в IG Farben, за да възпроизведе свойствата на найлон 6,6, без да нарушава патента за производството му. Това е полукристален полиамид. За разлика от повечето други найлони, PA6 не е кондензационен полимер, а вместо това се образува чрез полимеризация с отваряне на пръстена, което го прави специален случай при сравнението между кондензационните и добавъчните полимери. PA6 намира приложение в широка гама от продукти, изискващи материали с висока якост. Той е широко използван за производството на зъбни колела, фитинги и лагери, в автомобилната индустрия за части от долната част на корпуса и като материал за корпусите на електроинструменти.
- **PEEK** – Полиетертеретронът (PEEK) първоначално е разработен в края на 70-те години от американската авиокосмическа индустрия, която взима под внимание свойствата на материала на стабилност при високи температури и по този начин има потенциала за приложения с високо натоварване и висока температура. За съжаление днес цената на PEEK е все още твърде висока в сравнение с другите широко използвани полимери в зъбните предавки (175 лв./кг. срещу 5 лв./кг.).
- **UHMWPE** – Свръхвисокомолекулният полиетилен (UHMWPE, UHMW) е подмножество от термопластичния полиетилен. Известен също като високомодулен полиетилен (high-modulus polyethylene или HMPE) или високоефективен полиетилен (High-Performance Polyethylene или HPPE), има изключително дълги вериги, с молекулна маса обикновено между 3,5 и 7,5 милиона amu (atomic mass unit). UHMWPE се използва при производството на прозорци и врати от PVC (винил), тъй като може да издържи до топлината, необходима за омекотяване на материалите на основата на PVC, и се използва като формовъчен/камерен пълнеж за различни профили на PVC профили, за да могат тези материали да бъдат "*огънати*" или оформени около шаблона. UHMWPE се използва също при производството на хидравлични уплътнения и лагери, както и за зъбни колела. Той е най-подходящ за средни механични функции във водата, нефтената хидравлика, пневматиката и немодифицираните приложения. Той има добра износоустойчивост, но е по-подходящ за меки свързващи повърхности.
- **CNTs** – Въглеродните нанотръби (Carbon Nanotubes) имат най-висока работна температура, якост на опън и модул на еластичност, което ги прави идеални за зъбни колела, но те също имат и най-високата температура на топене, което ги прави трудни за производство и обработка, особено когато през 2016 г. са все още доста скъпи в сравнение с полимерите. Един от техните основните предимства е фактът, че те са самосмазващи се, което означава, че когато се използват CNTs не е наложително да се използва допълнителна смазка. Приложения им са предимно като добавки към полимери: Зъбни колела, велосипедни компоненти, ракети за тенис.

Предложението, което се прави в настоящата статия, е червячните зъбни предавки да бъдат изработени от композитни материали, създадени от смесването на гореизброените полимери с различно съдържание на въглеродни нанотръби.

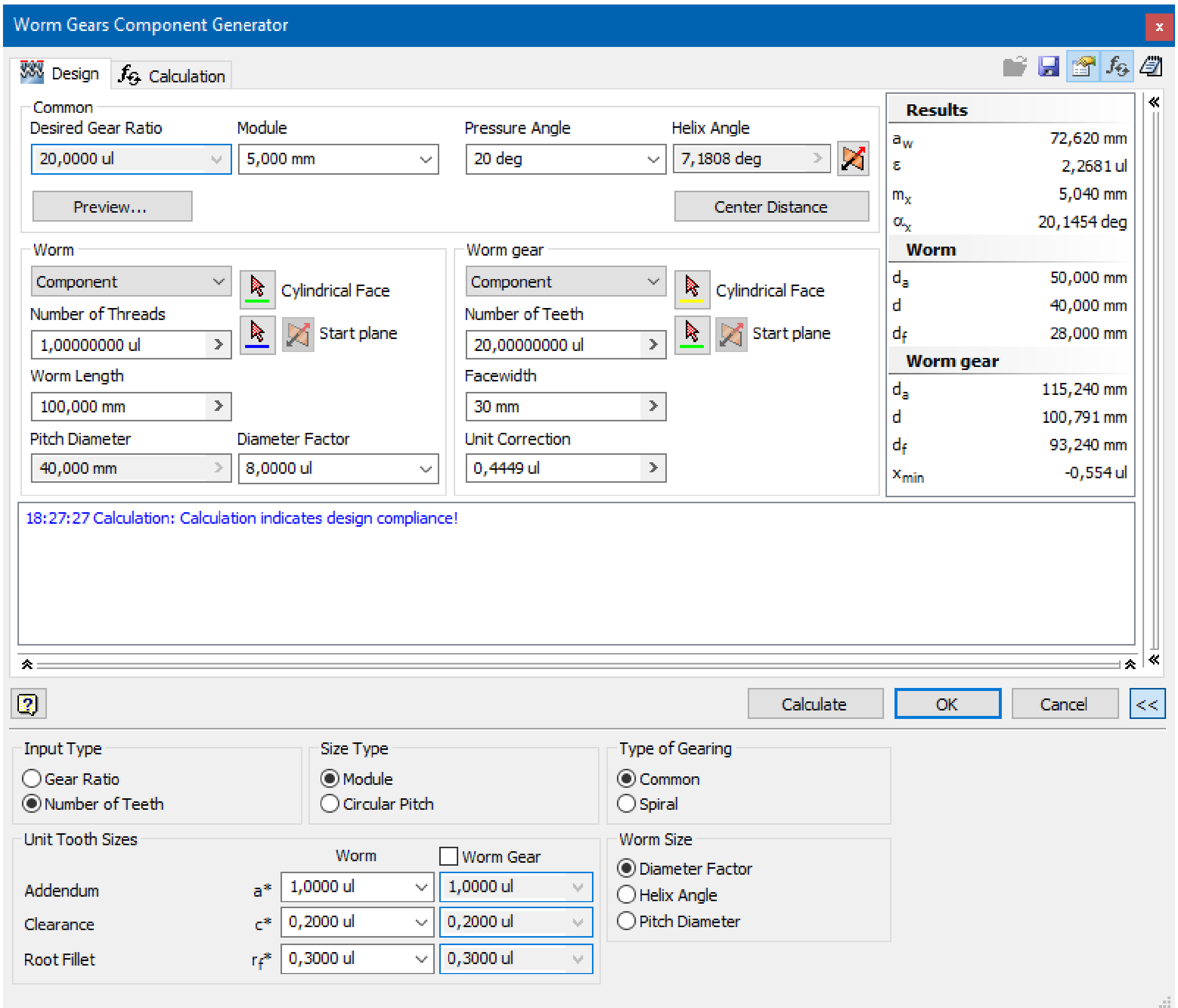
4. СОФТУЕРНА СРЕДА НА AUTODESK INVENTOR

Autodesk® Inventor® е програмна система от вида на тримерните, параметричните, базирани върху свойства или особености (features) системи за автоматизирано проектиране (CAD/CAM/CAE). В последните няколко години такива програми се превърнаха в основен инструмент за всяка организация, занимаваща се с проектиране и конструиране на машини и съоръжения, измествайки постепенно традиционните двумерни чертожни системи основно поради значително по-високата производителност и възможности за покриване на целия процес на разработка на изделията [2].

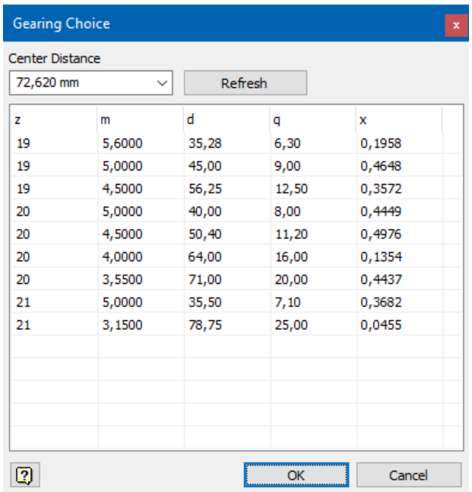
5. ПРОЕКТИРАНЕ НА ЗЪБНАТА ПРЕДАВКА. ВХОДНИ ДАННИ

Проектирането на двете еднотипни червячни предавки - ***M1*** и ***M2***, чрез които да се реализира движението на точка ***D*** (фиг.2, фиг.3) за покриване на диаграмата на работното пространство за движение, следва методиката, разработена и предложена системно и методично в [3].

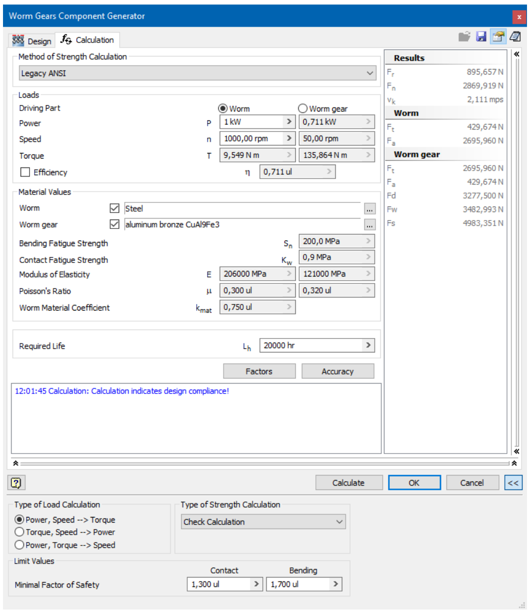
Геометричните, силовите и кинематичните параметри, необходими за проектирането на предавките при реализацията им с композитни материали, са показани директно в работния интерфейс на програмния продукт *Autodesk® Inventor®* (фиг.12 и фиг.13).



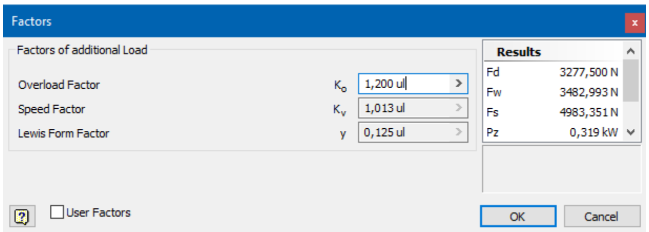
Фиг.12.а. *Задаване на геометричните параметри.*



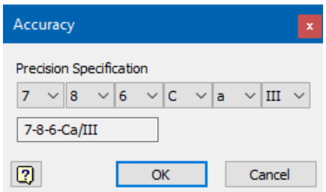
Фиг.12.б. Междуосово разстояние.



Фиг.13.а. *Въвеждане на силовите и кинематичните параметри на предавката.*



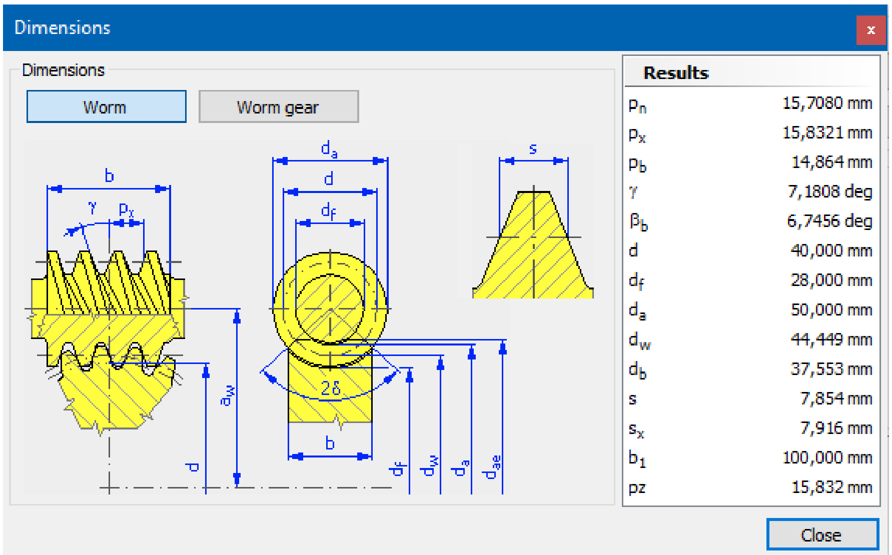
Фиг.13.б. *Коефициенти, отчитащи режимите на натоварванията.*



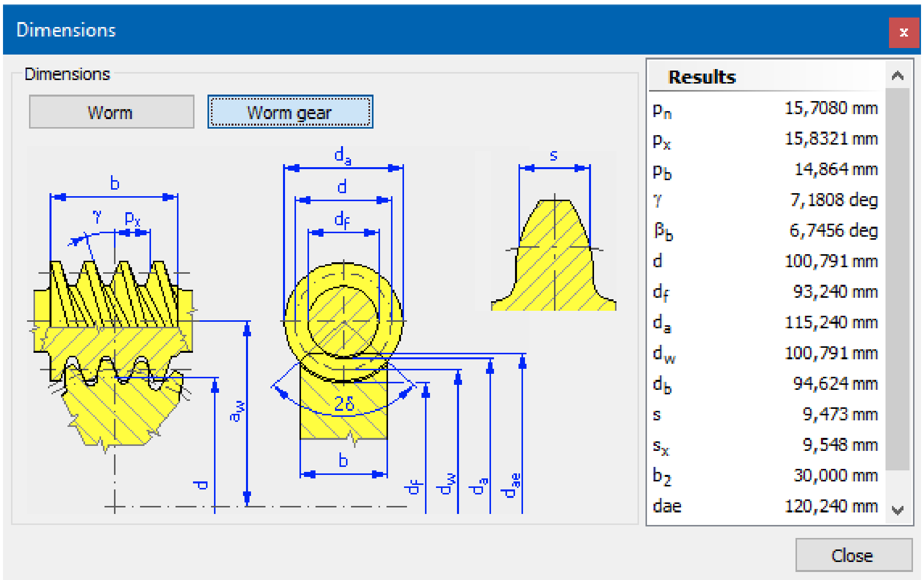
Фиг.13.в. *Клас на точност.*

6. КРАЙНИ РЕЗУЛТАТИ

Крайните резултати от проектирането на червяка и червячното зъбно колело, след активиране на командата *Calculate*, са показани в завършен вид на фиг.14 и фиг.15, които в последствие предстои да се отлеят или синтероват от гореизброените полимерни материали.

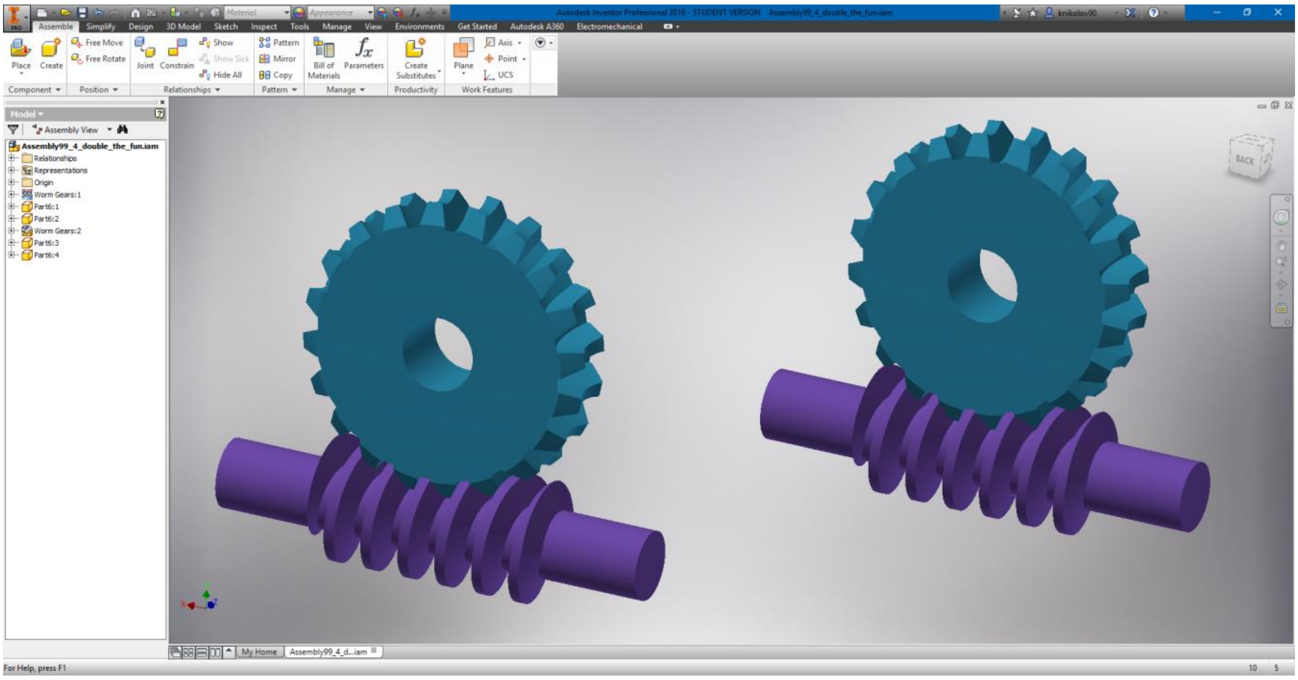


Фиг.14.а. *Предварителна изходяща зъбна геометрия с всички силови и геометрични параметри .*



Фиг.14.б. *Предварителна изходяща зъбна геометрия с всички силови и геометрични параметри.*

Крайният модел на проектираните в разработката червячни зъбни предавки за управлението на прототип на индустриален робот с паралелна кинематика и две степени на свобода е показан на фиг.15.



Фиг.15. *Двете еднотипни едноходови червячни зъбни предавки за управлението на индустриален робот с паралелна кинематика и две степени на свобода.*

ЗАКЛЮЧЕНИЕ

В настоящата работа:

- са проектирани еднотипни едноходови червячни зъбни предавки за линейно управление на движението на прототип на индустриален робот с паралелна кинематика, като са спазени предписанията и изискванията, систематизирани в литературата;

- червячните зъбни предавки са проектирани в софтуерната среда на *Autodesk® Inventor®* чрез използването на модула *Power Transmission*;
- тази предавки ще бъдат изработени от предложените полимерни материали в комбинация с въглеродни нанотръби с оглед олекотяване и подобряване на работата на предавките и привеждане на прототипа в класа на автономните роботи с паралелна кинематика.

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